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Urban spatial interaction

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Abstract. This paper centres on the development of a geometrical representation of movement and spatial interaction in urban areas, as distinct from the network representation commonly used in modern studies. All quantities are treated as distributions over geographic space, rather than concentrations at nodes of a network. We present a continuous model of spatial interaction and calibrate it for Greater Manchester. The model is a trip distribution model which produces an allocation of trips from any area to all other areas in accordance with a derived trip density function. The inputs to the model are density functions for residences and workplaces of car commuters, and a velocity field. The velocity field defines a measure of the travel time between different locations.

The outputs of the model are spatial distributions of accessibility of locations to jobs and residences, and a spatial distribution of traffic flow. We find that the location most accessible to jobs in Manchester is not in the centre of the city, but in a ring more than half a mile away from the centre. The maximum accessibility to residences is found to be approximately five miles away from the centre. The flow of traffic is derived as a spatial pattern and is found to peak approximately three miles from the centre of Manchester. The paper restricts itself to vehicular traffic and to radially symmetric spatial distributions. It should be seen as an attempt to introduce three major elements: time, distribution, and traffic assignment into a geometrical framework for dealing with problems of urban spatial interaction.

1 A geometrical representation of urban space

Any attempt to discern a spatial pattern in cities involves a large measure of abstraction. Students of spatial structure and spatial interaction are usually concerned with two types of abstraction: a geometrical representation, and a network representation. The distinguishing characteristic of the geometrical representation is that all the quantities which are discussed are associated with a *continuous* coordinate system, where every point is located in a unique position in geographic space.

The distinguishing characteristic of the network representation is that all quantities are associated with a *discrete* set of coordinates, where every point in geographic space is assigned to a particular zone and every zone is associated with one or more nodes in the network. Since the geometric representation involves the use of a continuous coordinate system, all findings can be described in terms of distributions of quantities in space. Many of the interesting findings in urban ecology, geography, and economics are of this nature. Park and Burgess identify five rings of changing land uses in the city (Park *et al.*, 1925). Clark (1951) approximates residential densities in cities by a negative exponential function. Alonso (1964) and others suggest a structure of rents declining with distance from the city centre. In all of these examples, quantities (or qualities in the case of land use) are associated with spatial coordinates. The pattern is discerned from the distribution of these quantities in space.

In the network representation such patterns cannot be discerned. The city is partitioned into a set of zones and the quantities associated with each zone are assigned to a node or a set of nodes. Hence nodes are assigned numbers or sets of numbers. These numbers do not form a spatial pattern, but are helpful in dealing with movement in urban areas. A generation of transportation studies has invariably adopted the network representation of urban space as a framework for analysis.

The major contribution of transportation studies to the field was the development and refinement of a variety of spatial interaction models. These models were used effectively even though the number of zones and links in the system was relatively large. Since the variables involved were all of a discrete nature, they yielded a fairly simple mathematical treatment, and a straightforward application of computers. Each study gave some insight into the particular structure of a given urban area, but the contribution of these studies to a general understanding of urban spatial structure and spatial interaction has been disappointing. Very few significant pattern properties of cities or of movement in cities have emerged.

The search for pattern properties leads us back to the geometrical representation of urban structure. The current weakness of the geometrical representation is the lack of a consistent theory of interaction in continuous two-dimensional urban space. Earlier studies of location theory have considered geographical space as a continuous terrain where movement proceeded in straight lines. Thus in classical location problems the optimal location of a parish school for children of three nearby villages is found by minimising the sum of straight line distances travelled by the children. While this may be an approximate representation in rural areas, it is far from satisfactory in the city. In urban areas straight line distances are not good indicators of travel cost or relative proximity. A mile in the centre of the city takes much longer to traverse than a mile on the periphery, due to the heavy concentration of traffic in central areas. A continuous theory of spatial interaction in cities must therefore concern itself directly with the variations in the velocity of travel in urban areas. We have chosen to treat velocities as continuously varying in space and independent of the direction of travel, and to describe travel as taking place along minimum time paths, bending away from regions of congestion. This approach has been described in a previous paper in this journal (Angel and Hyman, 1970a). The velocity field in this case functions as a continuous representation of the transportation system, in the same manner that the abstracted set of major road links functions as a transportation system in the network representation.

However, this does not yet tackle the problem of the continuous representation of spatial interaction: "how are trips, particularly commuter trips, distributed in urban space?" In the network representation this problem can be stated as follows: "how many work trips originating in a given zone terminate in a given zone, and how many vehicles travel through a given link?" In the continuous case we have to ask: "what is the density of trips between any two areas, and what is the spatial distribution of traffic flow?" In general the latter problem is more difficult, for while the first problem involves summations, the second involves integrations. It is possible, however, with very few simplifying assumptions, to construct a continuous model of urban spatial interaction.

This paper focuses on the construction of such a model and its application to a metropolitan area. It should be seen as a missing link in an integrated theory of urban space based on a geometrical representation of urban structure. All variables are treated as continuous distributions over the urban plane. Residences and work places are given as density functions over the plane. The system of streets is represented by a continuous scalar velocity field in the urban plane, where travel can proceed in any direction. Spatial interaction is represented as an interaction density, and is used for the derivation of continuous accessibility functions and a continuous distribution of traffic flow.

It was seen as particularly important to calibrate a continuous model of spatial interaction in an existing urban area, so as to test the relative applicability of the geometrical representation. The construction and calibration of the model was made possible with two major simplifications. To avoid the problem of modal split we have considered only people commuting to work by car, and to simplify the mathematical treatment we have regarded the density functions and the velocity field as radially symmetric functions, varying only with distance from the city centre. The model was calibrated for the Greater Manchester urban area for 1965, using data made available by SELNEC (South East Lancashire–North East Cheshire) Transportation Study[(1)]. A comparison between our results and those obtained by SELNEC indicates that the continuous model gives reasonable results. Since the computational and data requirements of the continuous model are small, it could become a powerful research tool for urban spatial analysis.

The remaining sections of this paper are devoted to the presentation of the continuous model of spatial interaction. Section 2 introduces the continuous trip distribution model. The model requires two kinds of inputs: the spatial distribution of residences and workplaces (section 3), and minimum travel times between locations (section 4). It produces two major kinds of outputs: the spatial distribution of accessibilities (section 5) and the spatial distribution of traffic flow (section 6). Each section incorporates theoretical development with experimental results.

2 A continuous model of trip distribution

Trip distribution theory concerns itself with relating individual travel behaviour to the overall pattern of travel. The theory centres around the derivation of a trip distribution model which predicts, from limited information, the number of trips between different locations in the city. These models have formed an integral part of all modern transportation studies, where their predictions are used to estimate the flow of traffic on the various links of a transport network. The theory has also been used to derive shopping models, residential location models, commodity flow, and various other topics in the field of human spatial interaction.

Traditional trip distribution models are all of a discrete nature, using a zonal system and a network representation of space. The main aim of this section is to present and interpret a *continuous* model of trip distribution which utilises a geometrical representation of urban space. We have chosen to introduce this model by familiarising the reader with the discrete trip distribution model derived by Wilson (1967)[(2)]. The discussion of the continuous model parallels exactly the discussion of the discrete model.

The discrete model

The city is divided into n zones. We assume that estimates of the number of trips originating in each zone have been made. These could be obtained from a trip generation model which estimates the expected number of work trips in a given area from population, occupation, and income characteristics of the area.

We also assume that the number of trip destinations in each zone has been estimated from employment characteristics of the area. Let O_i be the number of trip origins in zone i, and D_j be the number of trip destinations in zone j. It is

(1) We wish to thank A. Hawkins and M. Hammerstone of the Mathematical Analysis Unit in the Ministry of Transport, London, for their continuous assistance in obtaining and interpreting the data. The SELNEC distribution model is described in Wagon and Hawkins (1970).

(2) A similar model has been derived by Tomlin and Tomlin (1968). The ideas presented have also been developed by Schneider (1967), Cohen (1961), and others. For a more extensive introduction to the literature on trip distribution models the reader is referred to the April, 1970 (Volume 4, number 1) issue of *Transportation Research* devoted to an exposition of recent developments in this field.

conventional to assume that the total number of origins is equal to the total number of destinations[3]. Let T_{ij} be the number of trips between zone i and zone j. The total number of trips from i to all destinations is the number of trips originating at i:

$$\sum_j T_{ij} = O_i , \qquad i = 1, 2, ..., n . \tag{1}$$

The total number of trips terminating in a given zone j is the number of trip destinations in j:

$$\sum_i T_{ij} = D_j , \qquad j = 1, 2, ..., n . \tag{2}$$

In order to estimate T_{ij} we also need estimates of interzonal travel costs, c_{ij}, and the average expenditure on travel in the system, $\bar{c}$. We can then write

$$\sum_i \sum_j T_{ij} c_{ij} = T\bar{c} . \tag{3}$$

Wilson (1969) has shown that the most probable distribution of trips is obtained by maximising the entropy

$$H = -\sum_i \sum_j T_{ij} \ln T_{ij} \tag{4}$$

subject to the constraints (1), (2), and (3). This maximisation yields[4]

$$T_{ij} = A_i B_j O_i D_j \mathrm{e}^{-\beta c_{ij}} . \tag{5}$$

The constraints (1) and (2) can now be rewritten as

$$\frac{1}{A_i} = \sum_j B_j D_j \mathrm{e}^{-\beta c_{ij}} , \qquad i = 1, 2, ..., n , \tag{6}$$

and

$$\frac{1}{B_j} = \sum_i A_i O_i \mathrm{e}^{-\beta c_{ij}} , \qquad j = 1, 2, ..., n . \tag{7}$$

The above $2n$ equations together with equation (3) can now be solved for the A_i's, B_j's and β. These equations require an iterative solution. Once a solution has been obtained, equation (5) predicts the number of trips between every pair of zones.

Equation (5), together with equations (3), (6), and (7), form the trip distribution model. The model predicts that the number of trips between zone i and zone j is proportional to the number of origins in i and the number of destinations in j, and declines exponentially with the travel cost between i and j. The reciprocals of the balancing factors A_i and B_j have been associated with the accessibility of zone i to employment opportunities, and the accessibility of zone j to workers respectively. The balancing factors are functions of position only and are independent of zone size. The effect of doubling the *levels* of population and employment in every zone is to double the expected number of trips between each pair of zones. The effect of doubling the *area* of zones is to quadruple the expected number of trips between each pair of zones, rather than double them. This should clarify the discussion of this problem by Wilson (1967, p.253).

(3) The total number of trips in the system, T, is therefore given by

$$T = \sum_i O_i = \sum_j D_j .$$

It is worth noting that this assumption is not necessary to the derivation of the model (Hyman, 1969).

(4) The proof appears in Wilson (1969, pp.7-15).

The continuous model
When urban space is represented as a field, we need no longer consider a zoning system or a transport network. The distributions of population, employment, and transportation are given as areal distributions. The model presented here requires the continuous analogues of the inputs used in the discrete model: distributions of residences and workplaces, and a measure of travel time or travel cost between locations. These are then used to solve the equations of the model, and consequently to derive a measure of the density of trips between pairs of locations. In deriving the continuous trip distribution model, density functions for residences and workplaces of commuters are assumed to be given. These are discussed in greater detail and calibrated in the following section. Measures of travel time between locations are also assumed to be given. These are derived and discussed in section 4.

Consider a polar coordinate system (r, θ), where r is the distance from the city centre, and θ is the angle between the radius vector and a fixed axis. An element of area in this polar coordinate system is $r\,\mathrm{d}\theta\,\mathrm{d}r$[5]. We define the *origin density function*, $O(r_1, \theta_1)$, as the number of trips originating in a unit area about the point (r_1, θ_1). Similarly we define the *destination density function*, $D(r_2, \theta_2)$, as the number of trips terminating in a unit area about the point (r_2, θ_2). In this study we have adopted one square mile as our unit of area, but other unit measures may be adopted. The diameter of the unit area must be small enough to avoid missing significant variations in density, but large enough to average out local fluctuations.

The value of the *trip density function*, $T(r_1, \theta_1, r_2, \theta_2)$, is the number of trips from a unit area at (r_1, θ_1) to a unit area at (r_2, θ_2). Its dimensions are number/unit area/unit area. If $T(r_1, \theta_1, r_2, \theta_2)$ is defined for a given unit area, then doubling the unit area will quadruple $T(r_1, \theta_1, r_2, \theta_2)$ for every pair of points (r_1, θ_1) and (r_2, θ_2).

As in the discrete formulation, we assume that all trips take place during a specified time period, in our case the morning rush hour. It follows that the total number of trips from a given unit area at (r_1, θ_1) to all other areas is the density of origins at (r_1, θ_1):

$$\int_0^{\infty}\int_0^{2\pi} T(r_1, \theta_1, r_2, \theta_2) r_2 \mathrm{d}\theta_2 \mathrm{d}r_2 = O(r_1, \theta_1) \quad \text{for all } r_1, \theta_1 \,. \tag{8}$$

Similarly, the total number of trips originating in the entire city and terminating in a unit area at (r_2, θ_2) is the density of destinations at (r_2, θ_2):

$$\int_0^{\infty}\int_0^{2\pi} T(r_1, \theta_1, r_2, \theta_2) r_1 \mathrm{d}\theta_1 \mathrm{d}r_1 = D(r_2, \theta_2) \quad \text{for all } r_2, \theta_2 \,. \tag{9}$$

We still need a measure analogous to the travel cost between zones, c_{ij}, in the discrete model. Given the coordinates of any two points (r_1, θ_1) and (r_2, θ_2), we derive an estimate for travel time between these points, $t(r_1, \theta_1, r_2, \theta_2)$. Here travel time is taken to be a satisfactory proxy for travel costs, as only travel by car is considered. The estimates of travel time are obtained by constructing a velocity field (Angel and Hyman, 1970a) where speed is a smooth function of position. Travel times are then calculated along minimum time paths between any two points.

Given the average travel time in the system $\bar{t}$, we can write the third constraint equation as

$$\int_0^{\infty}\int_0^{\infty}\int_0^{2\pi}\int_0^{2\pi} T(r_1, \theta_1, r_2, \theta_2) t(r_1, \theta_1, r_2, \theta_2) r_1 r_2 \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}r_1 \mathrm{d}r_2 = T\bar{t} \,, \tag{10}$$

(5) The following model is presented in polar coordinates, but the model equations can be easily rewritten using plane cartesian coordinates (x, y). The polar notation was adopted for later use in the radially symmetric model.

where T is the total number of trips in the system. T must satisfy the equation:

$$T = \int_0^\infty \int_0^{2\pi} O(r_1,\theta_1) r_1 \mathrm{d}\theta_1 \mathrm{d}r_1 = \int_0^\infty \int_0^{2\pi} D(r_2,\theta_2) r_2 \mathrm{d}\theta_2 \mathrm{d}r_2 \,, \tag{11}$$

since the total number of trips, the total number of origins, and the total number of destinations are identical by definition.

In an analogous manner to the discrete formulation, the most probable distribution of trips is obtained by maximising the entropy

$$H = -\int_0^\infty \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} T(r_1,\theta_1,r_2,\theta_2) \ln T(r_1,\theta_1,r_2,\theta_2) r_1 r_2 \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}r_1 \mathrm{d}r_2 \,, \tag{12}$$

subject to the constraints (8), (9), and (10). This maximisation yields[6] a trip density function:

$$T(r_1,r_2,\theta_1,\theta_2) = A(r_1,\theta_1) B(r_2,\theta_2) O(r_1,\theta_1) D(r_2,\theta_2) \mathrm{e}^{-\mu t(r_1,\theta_1,r_2,\theta_2)} \,. \tag{13}$$

The constraints (8) and (9) can now be rewritten as

$$\frac{1}{A(r_1,\theta_1)} = \int_0^\infty \int_0^{2\pi} B(r_2,\theta_2) D(r_2,\theta_2) \mathrm{e}^{-\mu t(r_1,\theta_1,r_2,\theta_2)} r_2 \mathrm{d}\theta_2 \mathrm{d}r_2$$
$$0 \leqslant r_1 < \infty, \; 0 \leqslant \theta_1 \leqslant 2\pi \,, \tag{14}$$

and

$$\frac{1}{B(r_2,\theta_2)} = \int_0^\infty \int_0^{2\pi} A(r_1,\theta_1) O(r_1,\theta_1) \mathrm{e}^{-\mu t(r_1,\theta_1,r_2,\theta_2)} r_1 \mathrm{d}\theta_1 \mathrm{d}r_1 \,,$$
$$0 \leqslant r_2 < \infty, \; 0 \leqslant \theta_2 \leqslant 2\pi \,. \tag{15}$$

Equations (14) and (15) together with the constraint (10) can now be used to obtain the values for $A(r_1, \theta_1)$, $B(r_2 \, \theta_2)$, and μ. Since these integral equations do not generally possess analytic solutions, numerical methods must be used to compute values for these functions. The balancing factors, $A(r_1, \theta_1)$ and $B(r_2, \theta_2)$, are functions of location only, depending solely on the coordinates of the points. Thus any interpretation of the balancing factors in the continuous model must deal with attributes of location. (The form of the balancing factors, their relation to accessibility, and their interpretation in the continuous model are presented in section 5.)

The radially symmetric model

Obtaining values for minimum travel times in an arbitrary velocity field, $V(r, \theta)$, involves a generalization of the procedure described by the authors in an earlier paper (Angel and Hyman, 1970b) for radially symmetric fields. While a graphic solution in the non symmetric case can be obtained manually using Huygens' principle, a general algorithm that could be programmed to produce travel times between any two locations has not yet been developed. We have thus limited the following discussion of the continuous model of spatial interaction to radially symmetric cities, using radially symmetric density functions and a radially symmetric velocity field as inputs.

By restricting the analysis to radially-symmetric cities we follow the tradition of many urban analysts. We must distinguish carefully, however, between the various traditions of the radially-symmetric urban model. Urban ecologists and economists have shown a preference for the concentric ring model. Park and Burgess have developed such a model of urban growth, the city being divided into a system of five concentric zones (Park *et al.*, 1925). Alonso (1964) has applied Von Thunen's concentric ring theory of agricultural location to urban land economics, dividing a

[6] The proof appears in appendix 1 of Angel and Hyman (1971).

single centred city into concentric rings, of infinitesimal width, which are the equilibrium amounts of land consumed by residences and firms. Other urban economists, notably Wingo (1961), Muth (1961, 1969), and Mills (1967, 1969), follow the same tradition of the original concentric ring model. In all these models travel proceeds on radial routes to the city centre. The question of trip distribution never arises since all trips end at the central business district (CBD).

A second approach to radially symmetric cities can be found in the transportation literature, where an established body of work is devoted to circular cities. Analysts try to measure different attributes of transport behaviour, making various assumptions on routeing systems. Mean journey lengths on various routeing systems (Holroyd, 1966), traffic flow on radial and circumferential routes (Lam and Newell, 1967), minimum cost paths (Wardrop, 1969), and various other topics have been studied in this context. This approach allows for traffic to proceed in all directions, and does not limit itself to movement to and from a single centre. The assumption of radial symmetry seems to have been introduced by the need for geometrical simplicity, and by the transport analyst's preference for radial and circumferential routeing systems.

These transport studies seem to have remained isolated from the other analyses of urban spatial structure and spatial interaction. At the same time, the theoretical approach of urban spatial economics has not found its way into the transportation literature. In the model presented below we attempt to lay a foundation for combining developments in both fields into an integrated conceptual framework.

The circularity of the city is retained. Residences and jobs are assumed to be distributed throughout the city in a radially symmetric fashion. This view follows that of the urban economists mentioned above and the empirical models of residential densities advanced by Clark (1951) and others. Jobs are also distributed throughout the city in a radially symmetric fashion. This generalises the assumption that all jobs are located in the CBD. Since jobs and residences are available everywhere in the city, travel in our model must take place in all directions, as in the transport theorists' view, rather than to and from the city centre. We assume, however, that there are no routeing systems and that travel proceeds along minimum paths in a radially symmetric velocity field. This concept of transport is similar to, but more general than, that suggested by Wardrop. Again, symmetry is introduced by the need for geometrical simplicity.

The assumption of radial symmetry modifies the equations of the model presented above. We assume that densities are a function only of the distance from the city centre:

$$O(r_1, \theta_1) = O(r_1) , \tag{16}$$

and

$$D(r_2, \theta_2) = D(r_2) . \tag{17}$$

The expression for travel time, and the expression for the trip density are assumed to be invariant under rotations about the city centre and reflections in any axis through the city centre. Under these assumptions it can be shown that the balancing factors are independent of the angular coordinates, and the trip density function takes the form[(7)]:

$$T(r_1, \theta_1, r_2, \theta_2) = A(r_1)B(r_2)O(r_1)D(r_2)e^{-\mu t(r_1, \theta_1, r_2, \theta_2)} . \tag{18}$$

(7) The proof of this result appears in Angel and Hyman (1971), appendix 2.

The constraint equations (14) and (15) can now be written as

$$\frac{1}{A(r_1)} = \int_0^\infty \int_0^{2\pi} B(r_2)D(r_2)\mathrm{e}^{-\mu t(r_1,\theta_1,r_2,\theta_2)}r_2 \mathrm{d}\theta_2 \mathrm{d}r_2 \, , \qquad 0 \leqslant r_1 < \infty \, , \tag{19}$$

and

$$\frac{1}{B(r_2)} = \int_0^\infty \int_0^{2\pi} A(r_1)O(r_1)\mathrm{e}^{-\mu t(r_1,\theta_1,r_2,\theta_2)}r_1 \mathrm{d}\theta_1 \mathrm{d}r_1 \, , \qquad 0 \leqslant r_2 < \infty \, . \tag{20}$$

It follows that the expressions on the right hand side of equations (19) and (20) are independent of θ_1 and θ_2 respectively.

Given the above two equations, together with equation (10) which remains unchanged in the radially symmetric model, we can evaluate the decay parameter μ and determine the form for the balancing factors.

This model was calibrated for the Greater Manchester urban area, using data and results obtained by the SELNEC Transportation Study for 1965 [8]. The parameter μ was estimated and the balancing factors were derived as functions of distance from Central Manchester. (The forms for the balancing factors are presented and discussed in section 5.) In order to test the accuracy of our predictions of the distribution of trips, we have compared them with the results obtained by SELNEC. The relevant comparison is to compare the travel time distribution predicted by both models. Namely, how many trips of each duration are predicted by the models. This comparison is presented in figure 1. As can be observed from the figure, our model predictions follow closely those of SELNEC. Minor deviations can be detected. Our model predicts fewer short commuting trips and more long trips than the SELNEC model. It can be said, however, that given the simplifying assumptions discussed above, the continuous model yields satisfactory results.

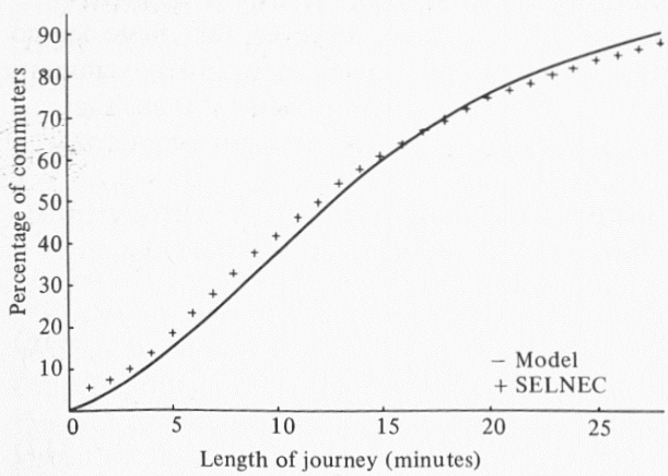

Figure 1. A comparison of travel time distributions of car commuters in the Greater Manchester urban area, 1965.

3 Residential and employment densities of commuters

Having presented the radially symmetric model, we now discuss the radially symmetric density functions for residences and workplaces which are used as inputs to the model. Traditional analyses often use the negative exponential decay function to

[8] The computer program for calibrating the continuous model appears in appendix 4 of Angel and Hyman (1971).

describe population densities:

$$d(r) = a\,\mathrm{e}^{-br}\ , \tag{21}$$

where a is related to the total population, b is a constant associated with the rate of decrease of densities with respect to r, the distance from the city centre. This analysis is originally due to Clark (1951), but has since been adopted and expanded by various analysts of urban spatial structure. Our preliminary analysis of the data showed that the exponential decay function did not fit the data properly. In the case of the distribution of residences the fit was worst near the city centre, where there are hardly any residences. Away from the centre residences quickly build up to a maximum density and then fall off again. This behaviour is also exhibited by Clark's data and Clark remarks that the negative exponential curve does not describe population densities in the CBD. In order to obtain a curve describing the data over the full range of distances from the centre we fitted a gamma distribution, with a density function of the form:

$$d(r) = ar^b\,\mathrm{e}^{-cr}\ . \tag{22}$$

The parameter b takes a positive value, so that the density of residences at the centre is zero.

In the case of jobs it was found that the cumulative distribution of workplaces approached the origin with a strictly positive gradient. This implies that the density of jobs at the centre is infinite and initially declines more rapidly than an exponential function. This behaviour is described by a density function also of the form of equation (22), where the parameter b takes a negative value.

Having chosen gamma distributions as models for the distributions of residences and jobs, we find the problem becomes one of estimating the parameters of these models. The procedure for obtaining the maximum likelihood estimates of these parameters is described in Angel and Hyman (1971), appendix 3.

The resulting density functions are

$$O(r) = 1164r^{0\cdot982}\mathrm{e}^{-0\cdot439r} \tag{23}$$

for origins or residences of commuters, and

$$D(r) = 4677r^{-0\cdot451}\mathrm{e}^{-0\cdot298r} \tag{24}$$

for destinations or locations of employment of commuters. The distribution curves for origins and destinations are presented in figure 2. In order to illustrate the radially symmetric properties of the density functions we also present them as surfaces in isometric form in figures 3 and 4[9].

In order to run the trip distribution model it was necessary to normalise the distribution of origins and destinations to the same total number. We notice, however, that at any radius the cumulative number of jobs always exceeds the cumulative number of workers. There are more destinations than origins within any circle about the city centre. The cumulative number of workers living within 20 miles of the centre of Manchester and commuting to work by car is found from figure 2 to be 166353. The cumulative number of jobs taken by car commuters up to this radius is 168698. We assume that all the workers within the 20 mile contour also work within this contour and taken together, their workplaces are distributed randomly amongst those available. We therefore reduced the density of jobs by the

[9] The isometric projections were obtained by means of a calcomp plotter available at the University of London Computer Centre. The subroutine for plotting the surfaces are developed by J. Adams, Royal College of Art. The furthest points on these figures, and in figure 5, are 20 miles away from the city centre.

ratio of these two numbers to obtain an equal total number of jobs and workers within the 20 mile contour.

The development of the general continuous model for the distribution of trips involves the relaxation of the radial symmetry assumption for population densities. Dacey (1968), for example, considers a city composed of several centres of population. He then assumes that the whole population can be divided into several classes, each class being distributed independently around one of the centres. The density function

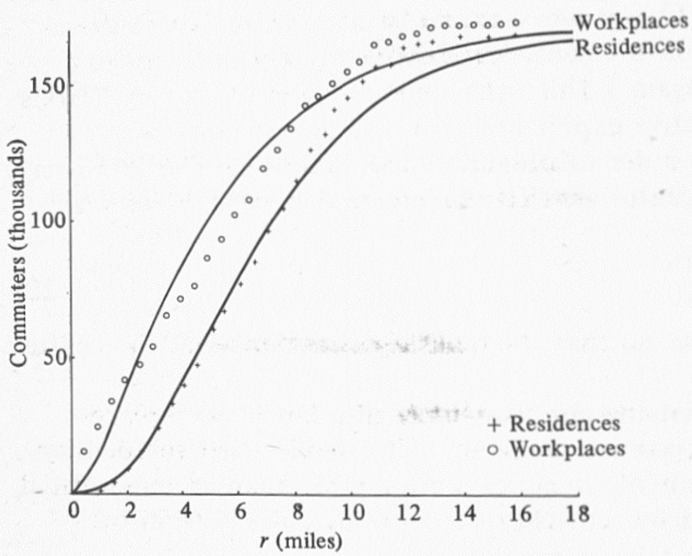

Figure 2. Residential and employment distributions of commuters in Greater Manchester, 1965.

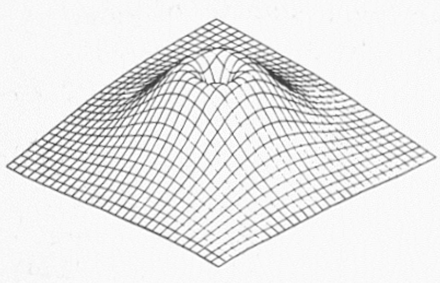

Figure 3. The density of residences of commuters in Manchester, 1965.

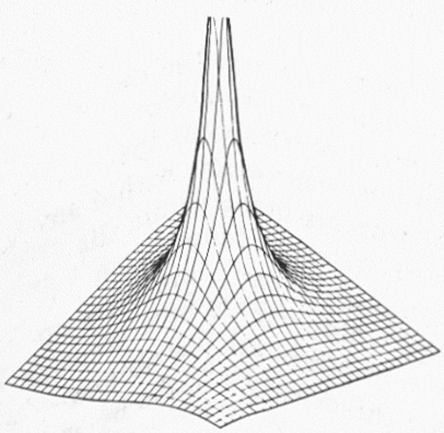

Figure 4. The density of employment locations of commuters in Manchester, 1965.

for the whole population is then obtained by adding together the density functions corresponding to each centre. This kind of analysis has an obvious application for modelling a regional distribution of population. It might be possible to associate centres of employment with different towns and decompose the distribution of residences by means of their employment location. Another kind of analysis is described by Gurevich and Saushkin (1966). They consider a single centre city where the population density declines steadily away from this centre. They then discuss five types of cities, each representing a different pattern of radial and angular variation in the population density. These density patterns may be relevant to describing the distribution of population in large conurbations.

4 Travel time on minimum paths

The continuous model presented earlier requires an estimate of the travel cost or travel time spent by commuters on their journey to work. When commuters reside and work throughout the city, we must have a procedure for calculating travel time or cost between any two points on the urban plane. The present section outlines this procedure for a radially symmetric city. A fuller discussion appears in Angel and Hyman (1970b).

Given the location of his residence and his job, the commuter must complete his journey to work. We assume here that the commuter selects the route which minimises the time spent on his journey. An analogy between travel time and expenditure is immediately apparent. Wingo (1961, p.43) pursues this analogy by indicating the similarity between expenditure on a commodity with a uniform price and travel time for uniform motion:

$$E = pq \tag{25}$$

and

$$t = \frac{1}{V}s\,, \tag{26}$$

where E is the expenditure, p the price, and q the quantity purchased, and where t is the travel time, $1/V$ the time per unit distance, and s the total distance travelled.

In commuting, where motion is no longer uniform due to congestion, the analogy must be pursued further. The marginal expenditure of a commuter at a point on his journey is the price he pays for consuming a unit of transportation services there, that is,

$$\frac{dE}{dq} = p\,. \tag{27}$$

By direct analogy we can write

$$\frac{dt}{ds} = \frac{1}{V}\,. \tag{28}$$

The latter equation can be recognised as the differential equation of motion. When we are only considering car commuters it is reasonable to assume that expenditures on the journey to work depend mainly on the time spent travelling. Petrol and other variable costs are assumed to vary directly with travel time. Thus total expenditure on commuting (including time spent) varies directly with travel time. Minimizing travel time will therefore be identical to minimizing expenditure on travel. We can then regard the price of a unit of transportation services, at each location, as the time spent in travelling a unit distance at that location. We proceed by analogy to rent theory.

In determining the location of his household the consumer perceives a rent at each location $R(r)$. This rent is the price per unit of housing (or land) at a given distance r from the city centre. The individual perceives the rent at a particular location, $R(r)$, as given, but $R(r)$ is determined by the interaction of the supply of housing and the aggregate demands of all consumers. Similarly, the commuter perceives $1/V(r)$ as given at each location although the velocity of travel, $V(r)$, is determined through the interaction of the supply of roads and other transport services with the aggregate demands of all commuters. Just as where many households desire to locate the rent is high, where many commuters desire to travel the time per unit distance is high, or the velocity of travel, $V(r)$, is low, which amounts to the same thing. When velocity is a function of where one travels in a city and is independent of the direction of travel we can construct a scalar velocity field, $V(r, \theta)$. In the radially symmetric case the velocity will be a function of distance from the city centre only, namely $V(r)$. In a previous paper we have constructed such a velocity field for Greater Manchester, 1965 (Angel and Hyman, 1970a, p.213). In a later paper we have found that our initial estimates for travel time on this velocity field varied directly with estimates for travel time on the transport network, but were consistently lower (Angel and Hyman, 1970b, p.10). A new velocity field was thus constructed for Greater Manchester. This velocity field takes the form

$$V(r) = 18{\cdot}5 - 12{\cdot}5\mathrm{e}^{-0{\cdot}56r} , \tag{29}$$

where r is measured in miles and $V(r)$ in miles per hour. Thus the average velocity at the centre is 6 mph increasing rapidly with distance from the city centre and reaching a maximum of 18·5 mph at large distances from the centre.

We can represent the behaviour of $1/V(r)$ by an *inverse velocity field* above the urban plane. The inverse velocity field for Greater Manchester, 1965, is presented in figure 5. As can be seen from the figure, the inverse velocity field is furthest from the urban plane at the city centre, and closest and parallel to the urban plane at large distances from the city centre. This field is then a measure of congestion in the city. It is high when congestion is high, and low when there is little or no congestion.

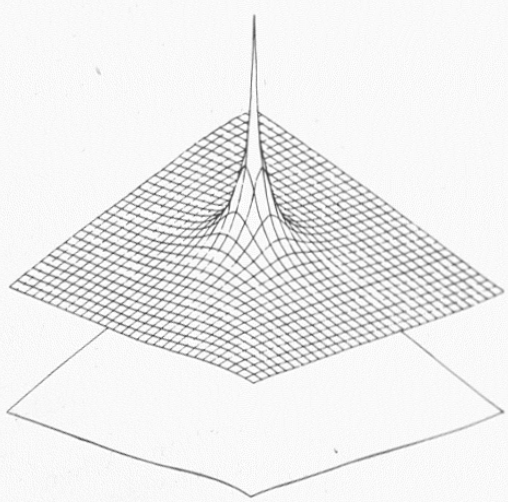

Figure 5. The inverse velocity field for Greater Manchester, 1965.

The total travel time along a path P on the urban plane is then given by the *path integral*

$$t_{\mathrm{AB}} = \int_{\mathrm{P}} \frac{\mathrm{d}s}{V(r)} \, , \tag{30}$$

and is represented as the vertical area between the inverse velocity field and the path P traversed in the urban plane. To define t_{AB} uniquely between two points we need to specify which route the traveller will take. We make the assumption that each traveller selects that route P which *minimises the total travel time* from his origin A to his destination B. In the discrete formulation, minimum travel costs are usually computed by means of a shortest route algorithm (Murchland, 1969). In the present formulation we solve the calculus of variations problem of finding the quickest path between two points and obtain a differential equation for the minimum time path.

This differential equation for radially symmetric cities has been derived in Angel and Hyman (1970a, p.217) and takes the form

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = \frac{r}{KV}(r^2 - K^2V^2)^{1/2} \, , \tag{31}$$

where K is a constant associated with the path. K is evaluated by integration of equation (31) and substitution for the polar coordinates of the points A and B. We can now substitute for the differential element of distance its expression in terms of the polar coordinate system:

$$\Delta s = (\Delta r^2 + r^2 \Delta\theta^2)^{1/2} \, , \tag{32}$$

and derive the differential equation for minimum travel time. This equation takes the form

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \frac{r}{V(r^2 - K^2V^2)^{1/2}} \, . \tag{33}$$

The integral form of this equation is

$$t_{\mathrm{AB}} = \int_{\mathrm{A}}^{\mathrm{B}} \frac{r\,\mathrm{d}r}{V(r^2 - K^2V^2)^{1/2}} \, . \tag{34}$$

We can thus compute travel time by first solving equation (31) for K and using this value in the integration of equation (34). A more detailed exposition of this method of calculation and a comparison between the values produced and those obtained by the SELNEC study for Greater Manchester may be found in Angel and Hyman (1970b). This procedure allows us to compute a unique measure of time $t(r_1, \theta_1, r_2, \theta_2)$ between any two points (r_1, θ_1) and (r_2, θ_2) in a given velocity field $V(r)$. We use these values to calibrate the radially symmetric trip distribution model described in section 2.

The development of the general continuous model for the distribution of trips requires the relaxation of the radial symmetry assumption for travel time. This symmetry arises from the assumption that the velocity of travel is a function only of the distance from the city centre. In Angel and Hyman (1970a) we solve the calculus of variations problem of finding the minimum time path between two locations and obtain Euler's differential equation as the condition for a minimum. This is a second order differential equation, but in the case of radial symmetry we can obtain a first order differential equation as its first integral. The first order equation is relatively easy to solve and the results of this paper and the detailed analysis in Angel and Hyman (1970b) arises from this solution.

In the general case we must deal with the second order equation directly. It should be possible to develop numerical techniques for solving this equation but the computation is bound to be far more laborious. Angel and Hyman (1970a) describe two other methods for the construction of minimum paths. The time surface method can be readily adopted in the non-radially symmetric case and a non-radially symmetric time surface constructed. The approach might then proceed through optimising techniques to produce the shortest paths on this surface. The third method—making use of Huygens' construction—is entirely independent of the radial symmetry assumption but needs considerable adaptation in order to become a practicable method for computation.

5 Balancing factors and accessibilities

This section is devoted to the presentation of the results of calibrating the radially symmetric trip distribution model for Greater Manchester, and to the interpretation of these results using basic concepts of spatial interaction. We begin by presenting the derived forms for the balancing factors $A(r_1)$ and $B(r_2)$, and studying their behaviour.

The reciprocal of the balancing factor for origins $1/A(r_1)$ is interpreted as the relative accessibility to job opportunities at a location r_1 miles from the city centre. $1/B(r_2)$ is interpreted as the relative accessibility of a location r_2 miles from the city centre to residence. In order to obtain unique measures of accessibility we derive the normalising factors, $A^*(r_1)$ and $B^*(r_2)$, following the approach of Kirby (1970). The reciprocals of the normalising factors are then interpreted as the accessibility of jobs and the accessibility to residence respectively. These are given for Greater Manchester, 1965.

The balancing factors

Recall equations (19) and (20) for the balancing factors in the radially symmetric model. For illustration purposes we rewrite these equations as

$$\frac{1}{A(r_1)} = \int_0^\infty B(r_2)D(r_2)r_2 \int_0^{2\pi} e^{-\mu t(r_1,\theta_1,r_2,\theta_2)} d\theta_2 dr_2 , \qquad 0 \leqslant r_1 < \infty , \tag{35}$$

and

$$\frac{1}{B(r_2)} = \int_0^\infty A(r_1)O(r_1)r_1 \int_0^{2\pi} e^{-\mu t(r_1,\theta_1,r_2,\theta_2)} d\theta_1 dr_1 , \qquad 0 \leqslant r_2 < \infty . \tag{36}$$

The following term of equation (35),

$$\int_0^{2\pi} e^{-\mu t(r_1,\theta_1,r_2,\theta_2)} d\theta_2 ,$$

can be interpreted as the proximity of r_1 and r_2. It will be large when $|r_1 - r_2|$ is small. The left hand term of equation (35) increases with the number of job opportunities available at r_2. $1/A(r_1)$ will therefore be large when there are many job opportunities in the proximity of r_1. By a similar argument $1/B(r_2)$ will be large when there are many residents in proximity to r_2. $1/B(r_2)$ is therefore a measure of the competition of all residents for jobs at r_2. The contribution of jobs at r_2 to the term $1/A(r_1)$ will therefore be small if there are many residents competing for jobs at r_2. $1/A(r_1)$ therefore has a natural interpretation as the relative accessibility to job opportunities at a location r_1 miles from the city centre. It is related directly to the number of jobs in close proximity, and is related inversely to the number of residents in close proximity to these jobs. By a similar argument $1/B(r_2)$ has the same interpretation of the relative accessibility to residents of a location r_2 miles from the city centre. It is related directly to the number of residents in close proximity, and is related inversely to the number of jobs in close proximity to these workers.

$1/A(r_1)$ and $1/B(r_2)$ are interpreted as relative values since, as can be seen by inspection of equations (10), (18), (19), and (20), they are both determined only up to a constant multiple, say k. Thus the balancing factors $kA(r_1)$ and $B(r_2)/k$ will yield the same distribution of trips. To obtain values for these functions, an initial value, say the value of $B(0)$, must be fixed arbitrarily. A form of the discrete trip distribution model which does not contain such an arbitrary scaling factor was developed by Kirby (1970) and is discussed below.

The normalising factors

Kirby defines the quantities A_i^* and B_j^* as the *normalising factors.* The normalising factors are constant multiples of the balancing factors A_i and B_j of the discrete model discussed in section 2. They appear when equation (5) of the trip distribution model is rewritten in the form

$$T_{ij} = \frac{A_i^* B_j^*}{\gamma T} O_i D_j \mathrm{f}(c_{ij}) , \tag{37}$$

where T is the total number of trips in the system and $\mathrm{f}(c_{ij})$ a decay function (taken to be the negative exponential function in the discrete model presented earlier). The values for the normalising factors are defined by the condition

$$\sum_i A_i^* O_i = \sum_j B_j^* D_j = \gamma T . \tag{38}$$

This condition, together with the constraints (1) and (2) which require the total number of origins, the total number of destinations, and the total number of trips to be identical, is sufficient to determine A_i^*, B_j^*, and γ. It can be verified that the relationships between the balancing factors, A_i and B_j, and the normalising factors, A_i^*, B_j^*, and γ, take the form

$$A_i^* = A_i \sum_j B_j D_j , \tag{39}$$

$$B_j^* = B_j \sum_i A_i O_i , \tag{40}$$

and

$$\gamma = \frac{1}{T} \sum_{ij} A_i B_j O_i D_j . \tag{41}$$

The definition of the normalising factors in the continuous model exactly parallels that of Kirby. The continuous version of equations (39), (40), and (41) produces expressions for $A^*(r_1)$, $B^*(r_2)$, and γ in the form

$$A^*(r_1) = 2\pi A(r_1) \int_0^\infty B(r_2) D(r_2) r_2 \mathrm{d}r_2 , \tag{42}$$

$$B^*(r_2) = 2\pi B(r_2) \int_0^\infty A(r_1) O(r_1) r_1 \mathrm{d}r_1 , \tag{43}$$

$$\gamma = \frac{4\pi^2}{T} \int_0^\infty \int_0^\infty A(r_1) B(r_2) O(r_1) D(r_2) r_1 r_2 \mathrm{d}r_1 \mathrm{d}r_2 . \tag{44}$$

The reciprocals of the normalising factors, $1/A^*(r_1)$ and $1/B^*(r_2)$, are respectively the accessibilities of a given residential location to job opportunities, and the accessibility of a given employment location to residences. These have been computed for the Greater Manchester urban area and are presented in figure 6.

The surprising feature of these results is that the maximum accessibility to jobs is not at the centre of the city but in a ring approximately a mile away from the centre, in spite of the large number of jobs available at the centre. This is due to the fact that congestion in the central area reduces the proximity of very central locations to jobs outside the centre. The inaccessibility of the centre to jobs for car commuters manifests itself clearly in the existing distribution of residences (figure 3), where it can be seen that there are hardly any residences of car commuters in close proximity to the centre.

The reader should also note that the accessibility of jobs to residences peaks quite a distance away from the centre. This is due in large measure to the concentration of residences in this region. It indicates that those industries and businesses seeking to maximise access to residences would benefit by moving away from the centre to suburban locations. As can be seen by comparing the existing distribution of jobs (figure 4) with the distribution of accessibility to residences, the current distribution of jobs fails to correspond to the distribution of accessibility. Most jobs are concentrated closer to the centre than the location most accessible to residences.

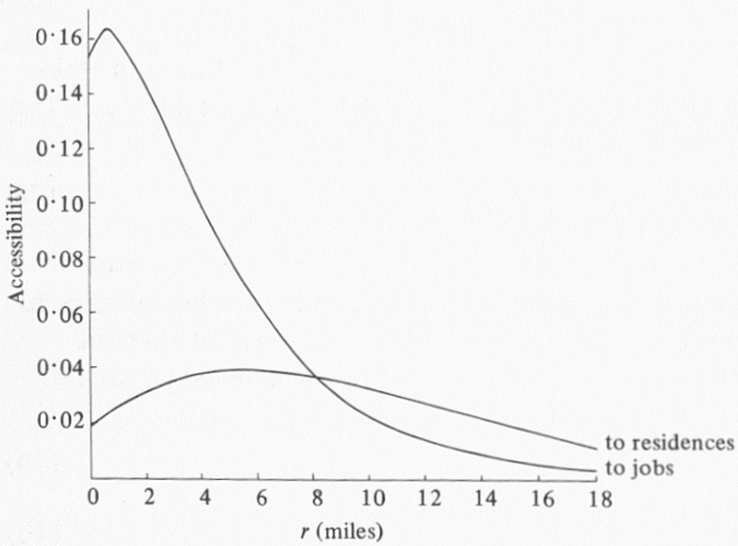

Figure 6. The accessibilities of residences to job opportunities and workplaces to residents in Greater Manchester, 1965.

6 Traffic flow

Traffic assignment

The problem of traffic assignment is to determine the total flow of traffic on each link in the road system. In the discrete formulation the trips between each origin-destination pair must be assigned to the minimum cost path, and the resulting loads on each link can then be aggregated. In the continuous formulation we do not have a network model of the road system. What we do have are the densities of trips between locations, and the minimum time path between them. So the problem of traffic assignment in the continuous formulation would be to aggregate the flow densities along each minimum path passing through a specified location. This problem has been formulated by Lam and Newell (1967). They assume that travel takes place only in radial and circumferential directions and derive a continuous approximation for flow of traffic in each of these directions. Lam and Newell do not derive a distribution of trips, assuming that this has been specified. In the present context we can use the output of our distribution model to produce the forms for these flows. In the following illustration we consider only the radial component of flow, $C(r)$, which is the number of trips crossing a circle r miles from the centre.

The function $C(r)$ is referred to here as the *cordon crossing function*. $C(r)$ is composed of three types of terms

$$C(r) = C_O(r) + C_I(r) + 2C_D(r) , \tag{45}$$

where $C_O(r)$ is the total number of trips crossing the circle in an *outward* direction only, $C_I(r)$ the number crossing only in an *inward* direction, and $C_D(r)$ the number which cross the circle twice, first in an inward direction and later in an outward direction. It has been shown in our previous paper (Angel and Hyman, 1970b, p.14) that the nature of minimum paths in the types of velocity fields usually encountered preclude the possibility of trips which cross a circle initially in an outward direction and later cross the same circle in an inward direction. These kinds of trips are therefore not included in the calculation of the cordon crossing function. The functions $C_O(r)$, $C_I(r)$, and $C_D(r)$ can be computed from the trip density function by means of the following integrals:

$$C_O(r) = \int_r^\infty \int_0^r \int_0^{2\pi} \int_0^{2\pi} A(r_1)O(r_1)B(r_2)D(r_2)\mathrm{e}^{-\mu t(r_1, r_2, \theta_1, \theta_2)} r_1 r_2 \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}r_1 \mathrm{d}r_2 , \tag{46}$$

$$C_I(r) = \int_0^r \int_r^\infty \int_0^{2\pi} \int_0^{2\pi} A(r_1)O(r_1)B(r_2)D(r_2)\mathrm{e}^{-\mu t(r_1, r_2, \theta_1, \theta_2)} r_1 r_2 \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}r_1 \mathrm{d}r_2 , \tag{47}$$

$$C_D(r) = \int_r^\infty \int_r^\infty \underset{|\theta_1 - \theta_2| > \overline{\theta}}{\int^{2\pi} \int^{2\pi}} A(r_1)O(r_1)B(r_2)D(r_2)\mathrm{e}^{-\mu t(r_1, r_2, \theta_1, \theta_2)} r_1 r_2 \mathrm{d}\theta_1 \mathrm{d}\theta_2 \mathrm{d}r_1 \mathrm{d}r_2 . \tag{48}$$

where $\overline{\theta}$ represents $\overline{\theta}(r_1, r) + \overline{\theta}(r_2, r)$.

In the first expression we compute the total number of trips from origins up to a radius r, and destinations that lie beyond r. These are all the outgoing trips crossing a circle of radius r. In the second expression we compute the number of trips between origins that lie beyond a radius r and destinations that lie inside r. These are all the ingoing trips crossing a circle of radius r. In the third expression we compute the number of trips between origins and destinations outside a radius r, which are separated by an angle wide enough so that the minimum time path between the two radii crosses the circle of radius r twice. $|\theta_1 - \theta_2|$ denotes the absolute value of the difference between θ_1 and θ_2. The function $\overline{\theta}(r_1, r)$ is the critical angle between the radii r_1 and r, and represents the angle traversed by a trip from r_1 whose path terminates at a tangent point to the circle of radius r. This variable and related variables associated with the geometry of minimum paths are described in more detail in Angel and Hyman (1970b).

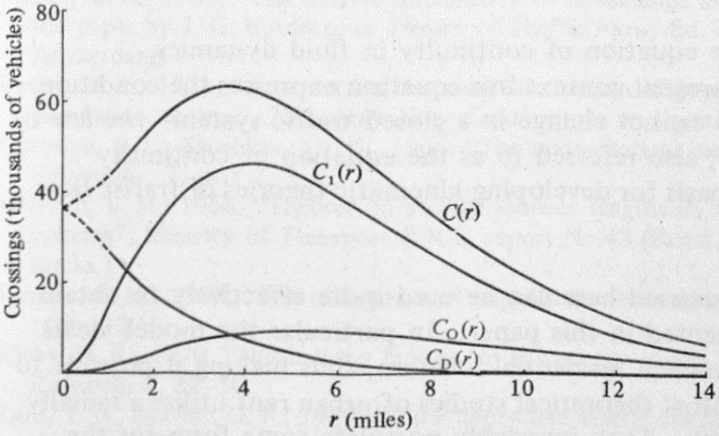

Figure 7. The crossing functions for Greater Manchester, 1965.

The forms for the crossing functions derived from the radially symmetric model for Greater Manchester, 1965 are presented in figure 7.

It is important to note two features in figure 7. First, it can be observed that the number of cordon crossings is influenced mostly by ingoing trips, and therefore peaks where these crossings are at a maximum, that is at about 5 miles from the centre of Manchester. The contribution of outgoing trips is minimal throughout. The contribution of double crossing trips is only apparent at close distances to the centre where they reach a value of approximately 30000 crossings at a mile from the centre. In other words, 15000 enter a circle of a mile radius and exit it before they reach their destinations.

Second, note that the curves for $2C_D(r)$ and $C(r)$ are shown in broken lines inside a mile radius. This is due to the form of the computer output which specifies crossings of radii at one mile intervals. We have therefore extrapolated these curves towards the centre, our only condition being that equation (45) is still satisfied. Consequently we cannot be entirely sure of the behaviour of the function $C(r)$ at the city centre. More precise calculations might indicate that this extrapolation needs to be revised.

These crossing functions describe the equilibrium distribution of traffic in the city. Their relationships to the flow of traffic on the roads at each location, and consequently to the velocity there, demand further study. When these are established it will be possible to use the continuous model to explore the consequences of changes in the capacity of the road system.

The law of conservation of traffic

An important relationship exists between the functions $C_O(r)$ and $C_I(r)$ which is quite independent of the distribution of trips. Define the net flow, $N(r)$, across the circle of radius r as the difference between the total ingoing flow and the total outgoing flow at r.

$$N(r) = [C_D(r)+C_I(r)]-[C_D(r)+C_O(r)] = C_I(r)-C_O(r)\ . \tag{49}$$

This difference must be equal to the difference between the total number of trips consumed inside the area bounded by the circle of radius r and the total number of trips produced inside that area, that is,

$$N(R) = 2\pi\int_0^R [D(r)-O(r)]r\,\mathrm{d}r\ . \tag{50}$$

The differential form for this equation is

$$\frac{\mathrm{d}N(r)}{\mathrm{d}r} = 2\pi r[D(r)-O(r)]\ . \tag{51}$$

Equation (51) is analogous to the equation of continuity in fluid dynamics (Rutherford, 1959, p.8). In the present context this equation expresses the condition that the total number of vehicles cannot change in a closed traffic system—*the law of conservation of traffic*. This law, also referred to as the equation of continuity (Ashton, 1966, p.39), provides a basis for developing kinematic theories of traffic flow.

7 Ideas for further research

The radially symmetric model discussed here can be used quite effectively to obtain several results which are not presented in this paper. In particular the model yields average travel times to work from each residential location, thus making it possible to analyse the economics of rent. Most theoretical studies of urban rent utilise a radially symmetric model for their analyses. They invariably postulate some form for the transport cost of commuting to work, as this cost, together with rent, determines the locational equilibrium of households. The average travel time function produced by this model sheds light on the economists' transport cost function, and will be discussed in a future paper.

The results presented in the last chapter on traffic flow can be studied further by utilising data on flow in existing cities, and by developing a spatial relationship between velocity and traffic flow, which will be consistent with empirical findings. Developments in this direction are crucial if the model is to be used for transport policy formulation.

All the results obtained in this paper rely heavily on radial symmetry, although the trip distribution model is presented for the general non-radially symmetric case. To relax the radial symmetry assumption one must represent the inverse velocity field as a landscape of hills and valleys, where congested areas are shown as peaks and fast corridors as troughs. A computer program must then be devised to calculate minimum travel time in such a field. This program can be structured on one of three principles: it can rely on the solution of the second order differential equation for the minimum time path; it can be developed by analogy to Huygens' construction of the path of a light ray in a continuous medium; or, it can be developed by analogy to the soap film model, where a soap film of minimum area forms between two vertical lines at the origin and the destination, the urban plane and the inverse velocity field. Each of these three procedures yields, in principle, the form of minimum paths and the value of the minimum travel time between any two points in a non-symmetric field.

A non-symmetric velocity field could be approximated from road speed data by averaging the velocity at different locations, and using a contouring program to obtain equal velocity contours in the city. The same method could be applied to obtain non-symmetric density functions for residences and workplaces of commuters. With the aid of these non-symmetric functions, the model could be run in its generalised form, possibly improving the results considerably and extending the applicability of the continuous model to policy research.

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Reviews

New towns by P. Merlin, Methuen, London, 1971, pp.276, £5.00

The term 'new town' must surely be one of the most loosely used labels in the terminology of planning. What exactly does it mean, what is implied in terms of content and function, and what are the typical circumstances which have led to the widespread adoption of 'new towns' as planning measures?

Professor Merlin's book sets out to answer these important questions in the context of nine very varied countries (UK, Sweden, Denmark, Finland, the Netherlands, France, USA, Poland, and Hungary) and for this reason alone the book seems assured of a wide and interested audience. Beyond this general readership, however, more specialized interest is likely to focus around two particular themes—urban design and its details, and the tools, mechanisms, and agencies of the planning process itself as it operates at what Hall (1970) has called regional/national and regional/local levels. For both these interests the book provides welcome detail, which is arranged under broadly consistent themes (design policy, building mechanism, housing, administration, finances, employment, population, social life, and so on) in the case of town construction and content and which is presented as a brief summary of the official planning structure and regional planning 'history' so far as the planning process is concerned.

Neither of these subjects is particularly easy to present. The first can too easily become a catalogue whilst the complexities of the latter are difficult enough to deal with in one's own language, let alone somebody else's, and it must be admitted that the book does not always escape the first of these pitfalls and is often rather overwhelmed by the second. On the design-detail side, comprehension is not helped by half-a-dozen town plans devoid of any key, or more fundamentally, the almost total absence of local maps *naming* the features described and illustrating their relationship to each other. If the reader's knowledge of suburban Stockholm or Upper Silesia is somewhat vague, or nonexistent, he will get little help from this book, and the many new place names will remain only dimly associated with real places and situations.

The more widespread difficulties encountered with the description of the planning process seem to arise partly from an attempt to deal too briefly with very complex situations, and partly in the translation from the original French text, for there are many passages which make difficult reading and some whose true meaning is quite obscure. In this manner, intriguing and important ideas such as the existence around Paris at one stage of a 'carcan' (though not translated, this means 'iron collar', was it a sort of urban fence?) are touched upon but not elaborated, and in the section on Britain, where the reviewer can judge more freely, one is often left with a feeling of a somewhat misplaced emphasis in the description of the new town and expanded town situation.

Even so this is a very difficult problem to tackle and the reader's sympathies are likely to be with Merlin in his pioneering attempt to present a rare comprehensive picture. After reading the book one scarcely knows whether to be appalled or reassured that in none of the countries described does this type of planning appear to be free from the confusion of plan, new plan, strategy, revised strategy, false starts, and fresh starts which are not unknown here—and Britain's record as described in the book emerges as well as any. Difficulties or not, there is much salutory reading here for 'regional planners'.

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References

Hall, P., 1970, *The Theory and Practice of Regional Planning* (Pemberton, London).

Land use by D. R. Denman and S. R. Prodano, Allen & Unwin, London, 1972, pp.240, £4.00

This is billed as an introduction to "proprietary land use analysis", that is, it purports to describe a discipline *sui generis*. While many of its inputs, as is so common in the social sciences, come from other disciplines, this reviewer finds most of the questions raised are economic, as one might