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URBAN TRAVEL TIME

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1. INTRODUCTION

To compute the travel time between locations in an urban area we have abstracted urban space into a velocity field, where travel takes place on the plane, where the velocity of travel varies continuously at different locations on the plane and where travellers move along minimum time paths. The movement of traffic then resembles the movement of a ray of light through a medium of varying density. This view of travel is not novel and has been discussed in the past by several authors such as Lösch [8], Beckmann [3], Bunge [4] and Warntz [13], [14]. The difficulty encountered in the treatment of movement in continuous space is usually of a mathematical nature. Studies remain generally suggestive and fail to produce the concrete observable results which the network models are capable of producing.

This paper, as our previous one, see Angel and Hyman [2], should be seen as a contribution to the analysis of space as a continuous terrain. This analysis is particularly suited to moden urban areas, where transportation networks are sufficiently dense to admit the consideration of the velocity at any location as an average velocity of travel on the road network in the immediate area surrounding this location. We thus ignore variations in travel speeds in different directions and stress that velocity is mainly a function of position rather than direction and is thus a *scalar* measure. Although we are aware that at a more detailed level speeds at any location do vary with direction, we attribute the major variations in speed to location. Thus suburban areas where densities are low allow for considerably higher velocities than congested urban centers. Congestion is taken to be contagious: if a few roads in a certain area are burdened with heavy traffic moving at low speeds then all roads in this area will in the long run become equally congested as drivers shift their minimum routes to reduce their travel time, and speeds in the area will become approximately identical. Transportation studies usually choose a series of major links in the road system to construct a road network, which is usually of a much lower density than the existing street pattern.[1] We choose to include all streets and to assume that the pattern is dense enough to admit the view of urban travel as taking place in a *continuous* field where velocities do not vary appreciably between nearby locations.

In the following analysis we restrict ourselves to *radially symmetric* velocity

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[1] The problem of calculating travel time is then reduced to the problem of finding the shortest route on the network. Procedures for finding shortest routes on networks have been extensively developed in the past decade. See Murchland [11].

fields, where velocity varies only as a function of distance from a single center. With further refinement the analysis could be extended to non-symmetric fields. The present analysis also implicitly restricts itself to surface travel. Travel time estimates can be obtained from urban velocity fields, assuming that travelers want to minimize the time of travel between their origins and destinations. When it is possible to compute travel time continuous space becomes more useful in the analysis of urban rent and shopping areas. Rent theorists, such as Alonso [1] and Mills [9] [10], utilize the continuous space view of transportation but are usually forced to simplify their concept of travel due to mathematical difficulties. Their analyses could be extended, in their own continuous framework, with a more realistic view of urban transport using velocity fields. Market area analysts, such as Huff [6] and Gambini, Huff and Jenks [5], also tend to limit their analyses to a simplified transport plane or grid. These could also gain a new dimension when the transport plane is distorted due to regular variations in velocity.

In order to test the validity of this view we have constructed a velocity field for the Greater Manchester Urban area. A scalar, continuous, and radially symmetric velocity field was estimated for Greater Manchester, 1965, by calculating the average link speeds at different distances from the center and fitting a negative exponential curve to the observations. The curve was found to be

$$V(r) = 24.9 - 16.9e^{-0.60r}$$

where r is the distance in miles from the city center and $V(r)$ is the velocity in m.p.h. at that distance. This velocity field is shown in Figure 1. Given this velocity field, we have utilized the concepts and procedures described below to measure

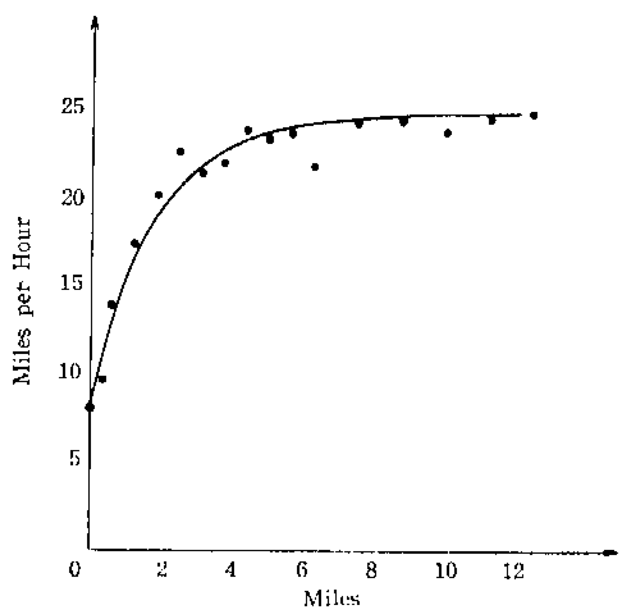

FIGURE 1. The Velocity Field for Greater Manchester, 1965

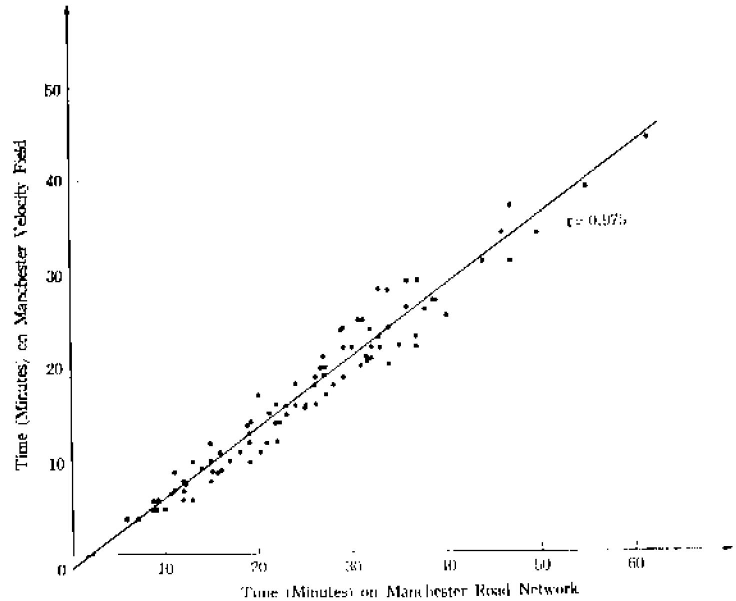

FIGURE 2. Comparison of Travel Time on the Greater Manchester Velocity Field with Travel Time on Shortest Routes on the Road Network

travel time on minimum routes between locations. We then compared our estimates of travel time with the estimates obtained by the SELNEC (South East Lancashire North East Cheshire) Transportation Study.[2] The latter were computed from survey data, utilizing a shortest route algorithm.

Ninety pairs of estimates were compared. The velocity field estimates were found to be roughly proportional to the network estimates, but were all found to be consistently lower than the network estimates (the slope of the regression line is 0.74). This is illustrated in Figure 2. The lower estimates may be due to the view of travel as taking place on curved paths, while in reality travelers wander through a system of streets and intersections. This notion suggests that all our velocities need to be scaled down by a factor which reflects the resulting increase in journey time. A new estimate for the velocity field for Manchester may now be obtained by multiplying all velocities by the slope of Figure 2, which is 0.74.

$$V(r) = 18.5 - 12.5\, e^{-0.55r}$$

This can only be an approximation since the regression line in Figure 2 does not pass through the origin. To obtain a better approximation for $V(r)$ a new sample of travel times can again be compared with time measurments on the new velocity field.

In the following sections we describe three interrelated methods for computing

[2] SELNEC (South East Lancashire North East Cheshire) Transportation Study: 1965 peak hour travel times between Study Zones. We wish to acknowledge the help of Michael Hammerstone of the Ministry of Transport Mathematical Advisory Unit in obtaining the data.

travel time on velocity fields. In all three we restrict ourselves to travel time along curved minimum routes, with no additional reference to the correction factor for computing real travel time. Further attempts to construct urban velocity fields and estimate travel time and correction factors will most likely improve these methods. If such methods prove reasonably successful, they could provide a simple and quick way of obtaining travel times --simple enough to be used by drivers and real estate agents worried about the time it takes to get from here to there. The only requirements for the estimation of travel time are a sample of velocities of travel at different distances from the center, a simple program for constructing a velocity field, and a simple program for measuring travel time on this velocity field. The analytical procedure for calculating travel time on curved paths on a given velocity field is presented in Section 2. A procedure for computing travel time, using time surfaces as analogues is presented in Section 3. A graphical method of estimating travel time with the use of a chronograph is presented in Section 4.

2. AN ANALYTICAL PROCEDURE FOR CALCULATING TRAVEL TIME

Given a continuous, scalar, and radially symmetric velocity field $V = V(r)$, we wish to calculate travel time on a minimum route between any two points (r_1, θ_1) and (r_2, θ_2) on the urban plane. This section sets the analytical framework for solving this problem. We have found it convenient to structure the discussion in the form of definitions, theorems and proofs. A complete summary of the procedure is presented in a flow chart in Figure 5. An element of distance on the plane is, in polar coordinates (r, θ),

$$\Delta s = \sqrt{\Delta r^2 + r^2 \Delta\theta^2}\,. \tag{1}$$

An element of time can be expressed in the form

$$\Delta t = \frac{\Delta s}{V} = \frac{1}{V}\sqrt{\Delta r^2 + r^2 \Delta\theta^2}\,. \tag{2}$$

The travel time on a path $r = r(\theta)$ from (r_1, θ_1) to (r_2, θ_2) is thus given by:

$$t(r_1, \theta_1, r_2, \theta_2) = \int_{\theta_2}^{\theta_1} \frac{1}{V}\sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta\,. \tag{3}$$

The path between two points on the plane which can be traversed in the smallest time is said to be the *minimum path.*

Theorem 1

A minimum path satisfies the differential equation

$$\frac{dr}{d\theta} = \frac{r}{KV}\sqrt{r^2 - K^2V^2} \tag{4}$$

where K is a constant associated with the path. The proof of this theorem appears in Angel and Hyman [2]. The differential equation, when solved for a given V, yields a second constant of integration. Both constants can be evaluated by requiring that the minimum path passes through the points (r_1, θ_1) and (r_2, θ_2). A *route* is a directed minimum path between two points. The *extension* of a route between two points is the union of all routes passing through the two points. K is said to be the *characteristic* of a route. K is also the characteristic of the extension of a route.

Theorem 2

Two routes have the same characteristic if and only if their extensions may be transformed into each other by a rotation about the center.

Proof: If two routes have the same characteristic K then, for each value of the radius r, their extensions must have the same value for $d\theta/dr$ by Equation (4). So for each radius the values of θ for the two extensions must differ by a constant. Thus one extension may be transformed into the other by a rotation about the center. Conversely if such a transformation exists then for each radius r the two extensions must have the same value for $r/kV\sqrt{r^2 - K^2V^2}$. Hence they must have the same characteristic K, and so the routes must have the same characteristics. The *minimum radius*, $r_{\min}(K)$, of a route with characteristic K is the shortest distance from the center of the city to the extension of the route. This is illustrated in Figure 3. Note that $r_{\min}(0) = 0$.

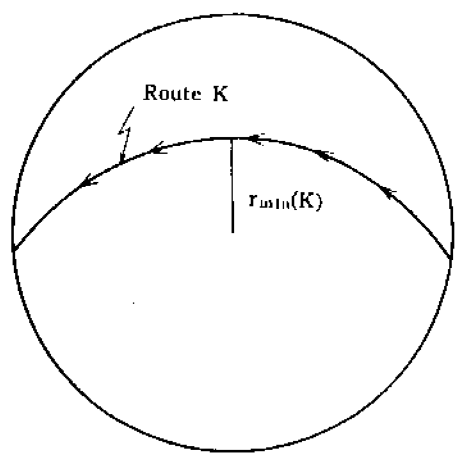

FIGURE 3. The Minimum Radius of a Route

Theorem 3

For velocity fields $V(r)$ with the properties that $r/V(r)$ is a monotonically increasing function of r, and

$$\lim_{r \to 0} \frac{r}{V(r)} = 0 . \tag{5}$$

The Equation

$$r = |K| V(r) \tag{6}$$

has the unique solution, $r_{\min}(K)$, for each characteristic K.

Proof: Suppose K is the characteristic of a route which does not pass through the center. The extension of this route must have a point of closest approach to the center. At this point $dr/d\theta = 0$. Solving Equation (4) for K when $dr/d\theta = 0$ yields

$$K = \pm \frac{r}{V}, \tag{7}$$

and hence Equation (6) is satisfied. Thus $r_{min}(K)$ is a solution to Equation (6). Since $r/V(r)$ increases monotonically with r this solution must be unique. If the route passes through the center it must have zero characteristic, by Equation (5). Thus Equation (6) will have only $r = 0$ as a solution.[8]

The *maximal characteristic*, $\bar{K}(r_1, r_2)$, is the positive characteristic of a route whose minimum radius is the smaller of r_1 and r_2. By Theorem 3 we have

$$\bar{K}(r_1, r_2) = \frac{\min(r_1, r_2)}{V[\min(r_1, r_2)]}. \tag{8}$$

The *critical angle*, $\theta(r_1, r_2)$, is the positive angular difference of a route between two radii r_1 and r_2 with a maximal characteristic $\bar{K}(r_1, r_2)$. This is illustrated in Figure 4. By Equation (4) we have

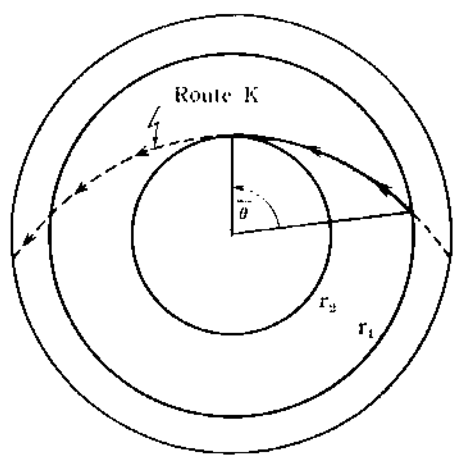

FIGURE 4. The Maximal Characteristic and Critical Angle of Radii r_1 and r_2

$$\frac{d\theta}{dr} = \frac{KV}{r\sqrt{r^2 - K^2V^2}}. \tag{9}$$

Integrating this expression between the smaller and the larger radii gives us an

[8] The analytic procedure is restricted by the conditions of Theorem 3 to those velocity fields which satisfy the conditions of the theorem. The requirement that $r/V(r)$ be a monotonically increasing function of r can be expressed in the form

$$\frac{V}{r} > \frac{dV}{dr} \quad \text{for} \quad 0 < r < \infty .$$

This requirement is satisfied for the Greater Manchester velocity field and will probably be met by most forms of velocity fields computed in existing urban areas.

expression for $\bar{\theta}(r_1, r_2)$, where $K = \bar{K}(r_1, r_2)$,

$$\theta(r_1, r_2) = \bar{K}(r_1, r_2)\int_{r_2}^{r_1} \frac{V\,dr}{r\,\sqrt{r^2 - \bar{K}^2(r_1, r_2)V^2}}\,. \tag{10}$$

The *angular difference*, θ_{12}, is the angle traversed by the route from an origin (r_1, θ_1) to a destination (r_2, θ_2).

θ_{12} may be calculated from the following expressions:

$$\left.\begin{aligned} \theta_{12} &= \theta_2 - \theta_1 && \text{for } -\pi \leq \theta_2 - \theta_1 \leq \pi \\ \theta_{12} &= \theta_2 - \theta_1 - 2\pi && \text{for } \pi < \theta_2 - \theta_1 \leq 2\pi \\ \theta_{12} &= \theta_2 - \theta_1 + 2\pi && \text{for } -2\pi \leq \theta_2 - \theta_1 < -\pi\,. \end{aligned}\right\} \tag{11}$$

θ_{12} is restricted to the range $-\pi \leqq \theta_{12} \leqq \pi$.

A *direct* route is a route which does not pass through its minimum radius. A *through* route is a route which passes through its minimum radius. It follows that the route between two points is a direct route if and only if the absolute value of their angular difference is smaller than the critical angle corresponding to their radii.

Define $\theta(r, K)$ to be the integral

$$\theta(r, K) = \int_{r_{\min}(K)}^{r} \frac{KV\,dr}{r\,\sqrt{r^2 - K^2V^2}} \tag{12}$$

where we take the positive root. Since r is always greater than $r_{\min}(K)$, $V > 0$ and $r > 0$, θ has the same sign as K. It can be verified that K is positive for counterclockwise routes and negative for clockwise routes. Clearly, $\theta(r, K)$ is the angle between the point nearest to the center on a route with characteristic K and a point on that route with a radial coordinate r. The angle $\theta(r, K)$ is measured in the direction of the route, and $\theta[r_{\min}(K), K] = 0$. Thus, for direct routes, we have

$$\theta_{12} = \int_{r_2}^{r_1} \frac{KV\,dr}{r\,\sqrt{r^2 - K^2V^2}} \quad \text{when } r_1 > r_2 \tag{13a}$$

and

$$\theta_{12} = \int_{r_2}^{r_1} \frac{KV\,dr}{r\,\sqrt{r^2 - K^2V^2}} \quad \text{when } r_1 > r_2\,. \tag{13b}$$

For through routes, we have

$$\theta_{12} = \theta(r_1, K) + \theta(r_2, K)\,. \tag{13c}$$

These equations establish the relationships between the characteristic K and the given angular difference of the origin and destination. By solving the appropriate equation we can compute the characteristic K of the route joining the two points.

Theorem 4

For travel time on a route with characteristic K we have

$$\frac{dt}{dr} = \frac{r}{V\,\sqrt{r^2 - K^2V^2}}\,. \tag{14}$$

Proof: From Equation (2) we obtain

$$\frac{dt}{dr} = \frac{\sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}}{V} . \tag{15}$$

On a route with characteristic K we have

$$\frac{d\theta}{dr} = \frac{KV}{r\sqrt{r^2 - K^2V^2}} . \tag{9}$$

Substituting for $d\theta/dr$ in equation (15) we obtain, for travel time on a route with characteristic K

$$\frac{dt}{dr} = \frac{r}{V\sqrt{r^2 - K^2V^2}} .$$

Define $t(r, K)$ to be the integral

$$t(r, K) = \int_{r_{\min}(K)}^{r} \frac{r\,dr}{V\sqrt{r^2 - K^2V^2}} \tag{16}$$

where we take the positive root. Since r is always greater than $r_{\min}(K)$, $t(r, K)$ must be positive. The term $t(r, K)$ is the minimum travel time from a point with a radial coordinate $r_{\min}(K)$ to a point with a radial coordinate r on a route with characteristic K. Clearly, $t[r_{\min}(K), K] = 0$. It can be easily verified that the minimum travel time between two points with radial coordinates r_1 and r_2 on a route with characteristic K is given for direct routes by

$$t_D(r_1, r_2, K) = \int_{r_1}^{r_2} \frac{r\,dr}{V\sqrt{r^2 - K^2V^2}} \qquad \text{for} \qquad r_1 < r_2 , \tag{17a}$$

and

$$t_D(r_1, r_2, K) = \int_{r_2}^{r_1} \frac{r\,dr}{V\sqrt{r^2 - K^2V^2}} \qquad \text{for} \qquad r_1 > r_2 , \tag{17b}$$

and for through routes by

$$t_T(r_1, r_2, K) = t(r_1, K) + t(r_2, K) . \tag{17c}$$

The preceding discussion provides us with a well defined procedure for calculating travel time between two points along a minimum route, in a given velocity field. We summarize this procedure in Figure 5.

We illustrate the procedure described above with an example. We assume that the central velocity in the city is zero and that the velocity then increases proportionally to the distance r from the center. The velocity field thus takes the form

$$V = \omega r . \tag{18}$$

Given two points (r_1, θ_1) and (r_2, θ_2), we want to compute the travel time on a minimum route between these points. First, observe that the given velocity field does not fulfil the condition of Theorem 3 since

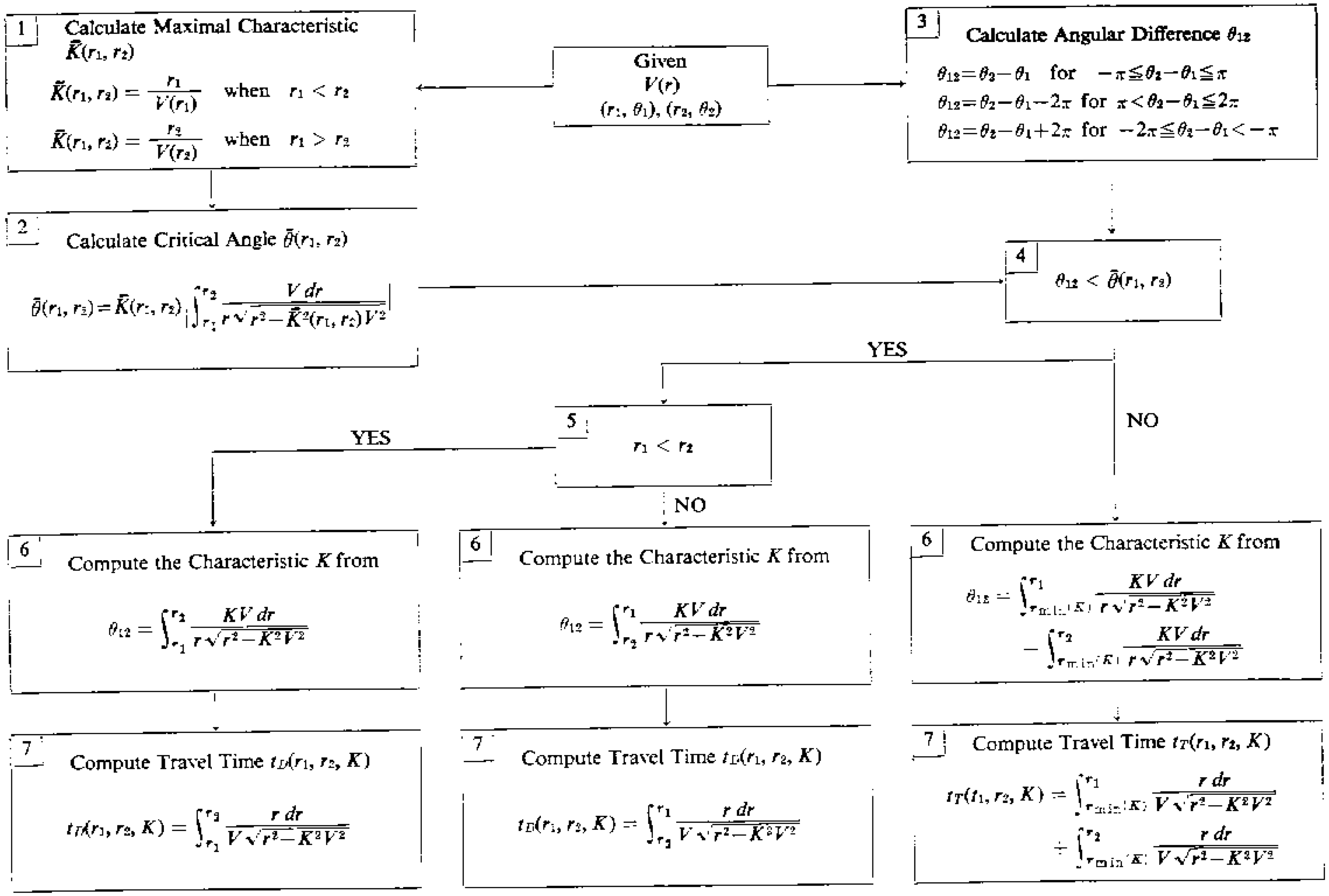

FIGURE 5. Procedure for Calculating Travel Time on Minimum Routes

$$\frac{r}{V(r)} = \frac{1}{\omega} = \text{const}\,. \tag{19}$$

The minimum paths in this velocity field are spirals which cut each radius at a constant angle.[4] Since each path reaches the origin it does not possess a minimum radius. We therefore need not calculate a maximal characteristic and a critical angle, since there are no through routes. We first compute the angular difference from Equations (11). Suppose $r_1 < r_2$. To obtain the characteristic K we must solve equation (13a) for K. Substituting for V, we obtain

$$\theta_{12} = \frac{K\omega}{\sqrt{1 - K^2\omega^2}} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{K\omega}{\sqrt{1 - K^2\omega^2}} \ln (r_2/r_1)\,. \tag{20}$$

To find an expression for the travel time we use the value of K from the above equation to solve equation (17a), which takes the form

$$t_0(r_1, r_2, K) = \frac{1}{\omega\sqrt{1 - K^2\omega^2}} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{1}{\omega\sqrt{1 - K^2\omega^2}} \ln (r_2/r_1)\,. \tag{21}$$

Substituting for K from equation (20), we obtain

$$t(r_1, r_2, \theta_{12}) = \frac{1}{\omega} \sqrt{[\ln (r_2/r_1)]^2 + \theta_{12}^2}\,. \tag{22}$$

This completes the procedure. In most velocity fields it will not be possible to obtain an analytical expression for travel time as a function of the given coordinates and the parameters of the velocity field. In these cases there is a need to develop computational methods for obtaining numerical values at each stage in the procedure.

3. THE TIME SURFACE METHOD

In our previous paper we have shown that if velocities form a continuous scalar and radially symmetric field, it is possible to transform the urban plane into a time surface. The time surface has the property that travel time between two points on the plane is identical with the distance between the images of these points on the time surface. Given a velocity field, we can form its time surface and compute travel time by measuring geodesics on that surface. We shall describe the transformation of the plane into a time surface, and compute travel time on the time surface for an example. An element of distance on the plane is, in polar coordinates (r, θ),

$$\Delta s = \sqrt{\Delta r^2 + r^2 \Delta\theta^2}\,. \tag{1}$$

An element of distance on the *time surface* is, in cylindrical coordinates (ρ, z, ϕ),

$$\Delta t = \sqrt{\Delta\rho^2 + \Delta z^2 + \rho^2\Delta\phi^2} \tag{23}$$

and this is illustrated in Figure 6.

[4] This result appears in Angel and Hyman [2; p. 220].

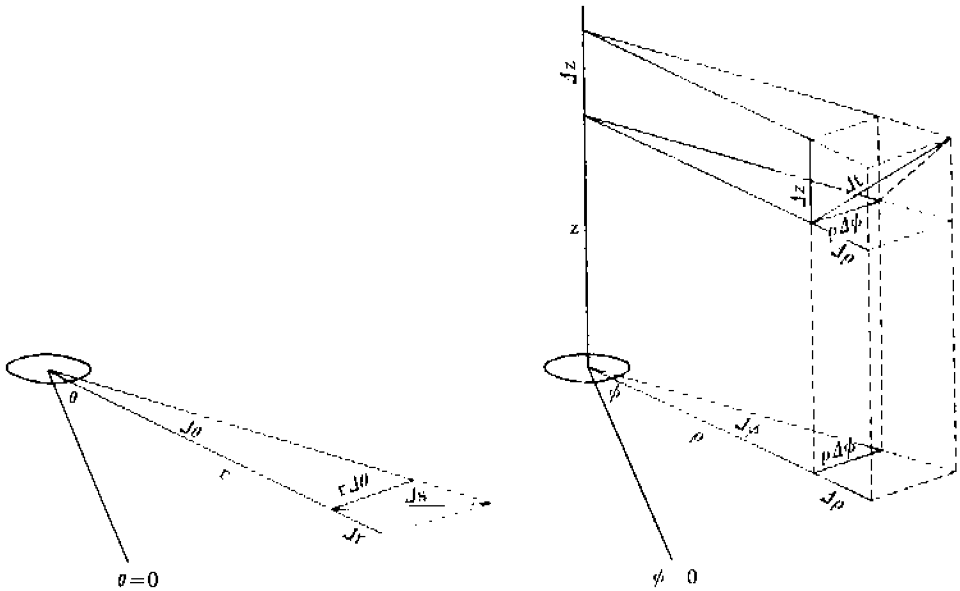

FIGURE 6. The Coordinate System (r, θ) of the Urban Plane and the Coordinate System (ρ, z, ϕ) of the Time Surface

Theorem 5

Let

$$\phi = \theta\,, \tag{24}$$

$$\rho = \frac{r}{V}\,, \tag{25}$$

and

$$z = \int \frac{1}{V^2}\sqrt{2rV\left(\frac{dV}{dr}\right) - r^2\left(\frac{dV}{dr}\right)^2}\,dr + C\,. \tag{26}$$

For any given C, these equations define a transformation

$$T_V:\quad (r, \theta) \rightarrow (\rho, z, \phi)\,.$$

The term T_V maps travel times between points on the urban plane onto distances on the time surface. The proof of this theorem appears in Angel and Hyman [2; p. 217] and is therefore omitted here. We illustrate the calculation of travel time on a time surface with a worked example.

Given the velocity field

$$V = \omega r^p\,, \tag{27}$$

where

$$0 \leq p < 1\,,$$

we wish to compute the travel time between two points (r_1, θ_1) and (r_2, θ_2).[6] We first show that the time surface is a cone. By Equation (25),

$$\rho = \frac{r}{V} = \frac{r}{\omega r^p} = \frac{1}{\omega} r^{1-p} . \tag{28}$$

By Equation (26)

$$z = \int \frac{1}{\omega^2 r^{2p}} \sqrt{2r\omega r^p p\omega r^{p-1} - r^2 p^2 \omega^2 r^{2p-2}}\, dr + C . \tag{29}$$

Letting

$$C = 0 ,$$

we then have

$$z = \frac{\sqrt{2p - p^2}}{\omega(1-p)} r^{1-p} = \frac{\sqrt{2p - p^2}}{1-p} \rho , \tag{30}$$

Thus z is proportional to ρ and the time surface is a cone. The distance R from the apex to any point on the cone can be evaluated from Equation (30) to yield

$$R = \frac{\rho}{1-p} . \tag{31}$$

We can now open the cone along one of its generators to obtain a pie-shaped area. The angle of opening of the pie-shaped area is $2\pi(1-p)$ as can be easily verified. Through the transformation of the cone into a pie-shaped area the angle ϕ is transformed into the angle α between two generators where

$$\alpha = \phi(1-p) . \tag{32}$$

Minimum paths on the pie-shaped area are straight lines, since the surface is a plane. The time between two points on this surface, (R_1, α_1) and (R_2, α_2), can be evaluated by the cosine rule. Let α_{12} be the angular difference of the two points. Then

$$t^2(R_1, R_2, \alpha_{12}) = R_1^2 + R_2^2 - 2R_1 R_2 \cos \alpha_{12} \tag{33}$$

This is illustrated in Figure 7. Substituting for the R_i values and α_{12} from Equations (31), (32), (24) and (25), we obtain an expression for travel time between the two original points on the urban plane:

$$t(r_1, r_2, \theta_{12}) = \frac{2}{\omega} \sqrt{r_1 + r_2 - 2r_1^{1/2} r_2^{1/2} \cos [(1-p)\theta_{12}]} . \tag{34}$$

Again, in many other velocity fields it would not be possible to compute an analytical

[6] This example is discussed by Wardrop [12]. Wardrop describes a method for calculating minimum paths and isochrones using conformal transformations with complex variables. He does not deal explicitly with time surfaces, but his analysis is, in fact, restricted to time surfaces which are developable into a plane (cylinders and cones). As we have shown in our previous paper, real time surfaces exist for all velocity fields where

$$0 \leq \frac{dV}{dr} \leq \frac{2V}{r} \qquad \text{for} \qquad 0 < r < \infty .$$

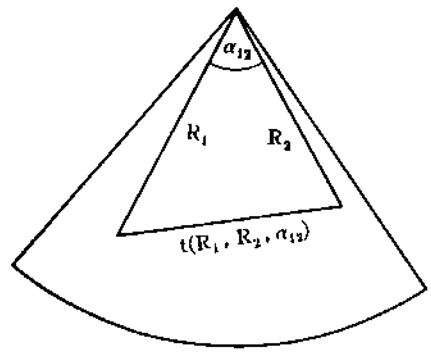

FIGURE 7. Travel Time on the Conic Time Surface

expression for travel time from the original coordinates. In some cases it may be feasible to construct a three dimensional model of the time surface and to measure distances between the images of points with a string.

4. A GRAPHICAL METHOD: THE CHRONOGRAPH

This procedure does not involve complex computations as those described in Section 2, nor does it make use of the time surfaces described in Section 3. Instead, we use a geometrical construction, analogous to that described by Huygens [7] in 1678 to determine the wave fronts of light. We use this method to construct a graph, the *chronograph*, which can be rotated about the center of the city to provide measures of travel time between any two points with a large degree of accuracy.

The chronograph for Greater Manchester was constructed by describing the set of minimum paths which are orthogonal to a given radial. To construct a minimum path from a given point on the radial we draw a circle of radius $V(r)\Delta t$ about this point, where r is the distance of that point from the city center. All points on this circle are Δt away from the original point. We now draw another set of small circles (or arcs) of radii $V(r)\Delta t$, all centered on various points on the first circle. We then draw the envelope of these circles. All points on this envelope are $2\Delta t$ away from the original point. The next set of circles is drawn about points of this envelope. By a repeated application of this method, we obtain a set of isochrones at intervals Δt from the original point. The minimum paths can now be drawn by tracing a curve through the original point, orthogonal to the original radial and to each of the isochrones. A selected set of minimum paths, orthogonal to this radial, is drawn by repeating this procedure for various points on the radial. The time contours can now be drawn by connecting the points of intersection of each minimum path with its appropriate isochrone $n\Delta t$. The time contour will then describe the set of points which are $n\Delta t$ away from the original radial.

We now have a completed chronograph. The chronograph for Greater Manchester is presented in Figure 8. The time contours are drawn at two minute intervals from the original radial. This chronograph was drawn for the velocity field

$$V(r) = 24.9 - 16.9r^{-0.50} .$$

It has the property that the minimum paths orthogonal to the original radial reach a limiting angle. This is reached by the path whose minimum radius is 0.56 miles. They then begin to straighten out again. The minimum path into the city center is thus a straight line orthogonal to the original radial. Travel time for trips on a path which comes within 0.56 miles from the center must therefore be measured on the dotted lines in Figure 8. To obtain readings of travel times between two points, we rotate the chronograph about the center of the city until the two points lie on one minimum path, or in a band between two minimum paths. We then simply add or subtract the times from the original radial to each of the points to obtain the correct travel time.

A chronograph drawn by hand in the above method was used in estimating travel times on the Manchester velocity field which are presented in Figure 2. It

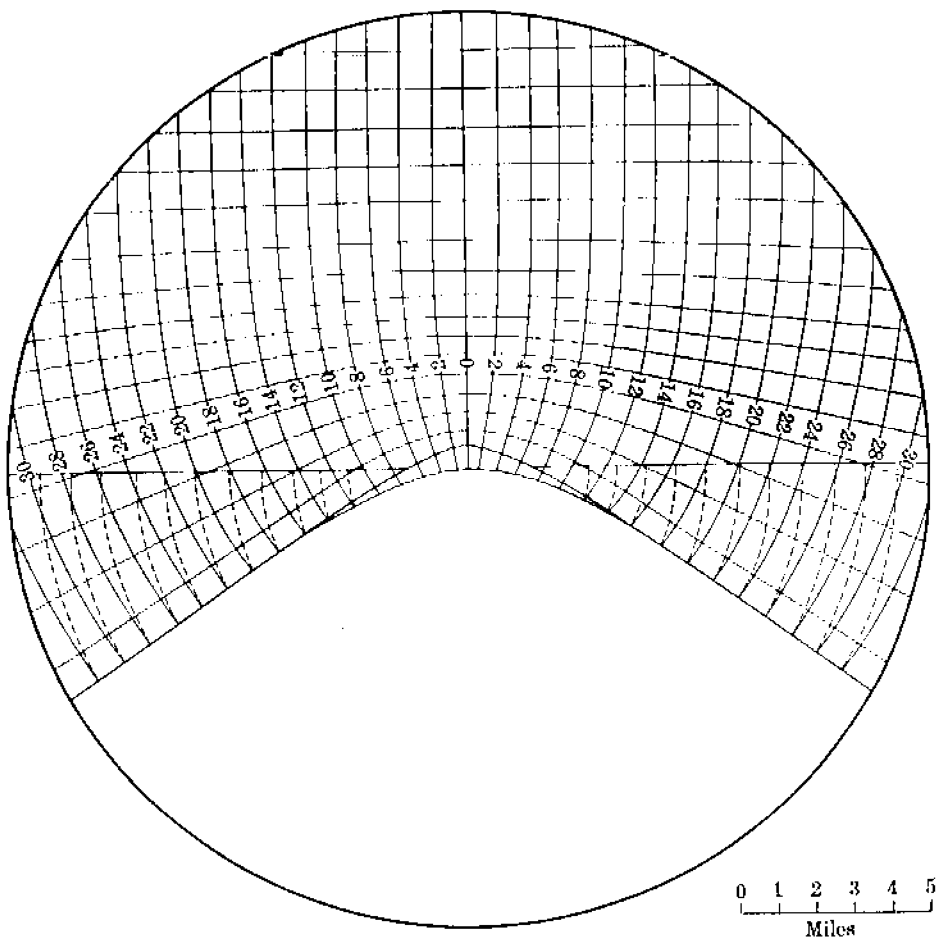

FIGURE 8. The Chronograph for Greater Manchester, 1965

can be easily seen from Figure 2 that these estimates given a good fit to the SELNEC estimates, although they are all consistently lower than those estimates as mentioned above. Utilizing the procedure described in Section 2 it is possible to construct a more accurate chronograph with the aid of the computer. The chronograph presented in Figure 8 was drawn in such a manner. A program was written to compute points along the minimum paths from the original radial and points of intersection of these paths with the time contours.

We have no tool at present for estimating travel time in urban areas, although the question of, "How long will it take to get there?" confronts everyone daily. Geographers and cartographers, although perfectly willing to commit themselves to road distances and street names, generally refrain from committing themselves to travel time estimates. The common reasoning is that times are so dependent on congestion. But then congestion seems to be here to stay and displays quite regular characteristics. The problem is to obtain time estimates taking urban congestion as a starting point. The analytical procedure can be used for developing transportation models which do not require a detailed description of the network. The time surface can be applied to generalizing the methods of location theory. The chronograph can be used by anybody to obtain rapid and reliable estimates of travel time in an urban area.

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