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Non-Compactness and Voter Exchange; Towards a Constitutional Cure for Gerrymandering
Non-Compactness as Voter Exchange: 
Towards A Constitutional Cure for Gerrymandering

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Non-Compactness as Voter Exchange: Towards A Constitutional Cure for Gerrymandering

The best way to protect the gerrymander is to deny the measurability of compactness. [John K. Wildgen]

Abstract

This essay proposes a novel compactness standard “for courts to identify at least the worst cases of gerrymandering and to provide a remedy.” This standard is based on the perception of gerrymandering as aggressive voter exchange, an exchange that distorts the shapes of election districts and renders them non-compact by expelling voters living close by and exchanging them for voters living further away. Aggressive voter exchange involves discrimination against large numbers of voters—possibly a majority of voters in some districts—only because of their voting preferences or their minority status. This form of discrimination, we argue, is prohibited by the First and the Fourteenth amendments. The proposed standard requires that a majority share of the area of any election district be contained in the Equal-Land-Area Circle, a circle about the center of

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the district that has the same land area in the state as that of the district. 7 percent of district shapes in 1996 and 10 percent in 2004 did not conform to this standard.

Introduction: A Proposed Standard for Detecting Impermissible Gerrymandering

The central objective of this essay is to propose that the U.S. Supreme Court adopt a simple and precise judicial standard “for courts to identify at least the worst cases of gerrymandering and to provide a remedy,” as demanded by Justice Souter. This standard is based on the perception of gerrymandering as aggressive voter exchange, an exchange that distorts the shapes of election districts and renders them non-compact. Voter exchange, as perceived by the Justices, involves expelling voters living close by and replacing them with voters living further away, for example, “A district that reaches out to grab small and apparently isolated minority communities is not reasonably compact.” The non-compactness that results from such voter exchange is easy to identify and measure, and the way we propose to measure it can be effectively grounded in the Constitutional harms that are created by aggressive voter exchange. Such harms involve discrimination against large

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numbers of voters—possibly a majority of voters in some districts—only because of their voting preferences or their minority status. This form of discrimination, we argue, is prohibited by the First and the Fourteenth amendments.

The essay is divided into two parts. In Part I, following this introduction, we introduce and discuss the legal foundation of the perception of voter exchange as a Constitutional harm. In section 1, we introduce the involvement of the Supreme Court in gerrymandering cases. In section 2, we focus on the connections between gerrymandering, voter exchange, and non-compactness. In Sections 3-5 we discuss voter exchange as a First Amendment harm, a Fourteenth Amendment harm, and the appearance of non-compactness as an Expressive harm. In Part II we introduce a theoretical foundation for the appropriate measurement of district compactness in the light of perceiving gerrymandering as voter exchange. In Section 1, we make the necessary distinctions between relevant and irrelevant measures of compactness. In Section 2, we focus on measures that correlate with ordinary people’s perception of the compactness of district shapes. In Section 3, we introduce measures that take into account local geographical constraints. In a concluding section, we introduce a standard for detecting gerrymandered districts that can consistently identify and constrain impermissible gerrymandering. The essay is followed by two annexes. In Annex I we examine Justice O’Connor’s observation that political gerrymandering is self-correcting and introduce evidence to show that this observation may be wrong. In Annex II we present the results of our survey of the perception of district compactness, showing that some measures better correlate with people’s perceptions than others.
The proposed standard for detecting and constraining impermissible gerrymandering is a general standard that can and should be applied to all redistricting efforts, be they partisan, bi-partisan, or involving the formation of minority-majority districts in accordance with the Voting Rights Act. We follow Justice Stevens in treating all gerrymandering as special cases of political gerrymandering: “In the line-drawing process, racial, religious, ethnic and economic gerrymanders are all species of political gerrymanders.”

In Vieth v. Jubelirer, Justice Stevens elaborates this contention further, requiring that political gerrymandering be treated in the same way that racial gerrymandering is treated by the Court:

> In my view, the same standards should apply to claims of political gerrymandering, for the essence of a gerrymander is the same regardless of whether the group is identified as political or racial.

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4 The Voting Rights Act allows for the creation of districts in which a minority forms a majority, so as to prevent the dilution of minority votes: “The extent to which members of a protected class have been elected to office in the state or political subdivision is one circumstance which may be considered” in assessing a dilution claim. 42 U.S.C. § 1973(b) (1988). The Supreme Court, in Thurnburg v. Gingles required that to qualify as a minority-majority election district, the minority community must be “sufficiently large and geographically compact.” Thurnburg v. Gingles 478 U.S. 50-51 (1986). The obligation to create such districts does not, therefore, override the required adherence to a compactness standard, were such a standard to be adopted by the Court.

Gerrymandering always involves the drawing of district boundaries to maximize the voting strength of the dominant political faction and to minimize the strength of one or more groups of opponents.

Thus, the critical issue in both racial and political gerrymandering is the same: whether a single nonneutral criterion controlled the districting process to such an extent that the Constitution was offended.\footnote{Vieth v. Jubelirer, 541 U.S. 335-336 (2004) (Stevens, J., dissenting).}

We are thus searching for a single standard to restrain all forms of gerrymandering. The adoption of such a standard by the Court may not eliminate gerrymandering completely, but it should clearly identify and prohibit impermissible gerrymandering, no matter what interests it may serve or what objectives it may seek to accomplish. Our approach to setting this standard thus follows Pildes and Niemi’s interpretation of the Court ruling in \textit{Shaw v. Reno}:

Geography and interest are both permissible grounds for constructing election districts, as long as the districting process is not reduced to a single-dimensional process in which interest appears to dominate overwhelmingly…. \textit{Shaw} rests on the view that, in certain areas, the Court’s role in construing the Constitution should be to require policy makers to accommodate and sustain the tension between conflicting...

The proposed standard introduced in this essay is based on a simple, yet novel, compactness measure that overcomes many of the inherent limitations of earlier measures. Compactness is quantified in a way that relates it directly to the Constitutional harms perpetrated by aggressive voter exchange. Simply stated, the proposed standard requires that a \textit{majority share} of the area of any election district be contained in the Equal-Land-Area Circle, a circle about the center of the district that has the same land area as that of the district (see figure 2). That circle is taken to be the hypothetical shape of the district in the absence of gerrymandering; the \textit{natural} shape that the district will have taken if it were maximally compact. In 2004, for example, 44 out of 435 district shapes (10\%) did not conform to this standard, up from 30 in 1996 (7\%), and all these districts, as we shall show later, have been aggressively gerrymandered.

It should be stated at the outset that our proposed standard identifies \textit{most}, but not all, aggressive gerrymandering. That is, some district shapes that appear to have been aggressively gerrymandered are not rendered impermissible by it. It is therefore an \textit{asymmetric} standard: it rules out district shapes that have less than a majority share of their areas in their Equal-Land-Area Circle as impermissible, but it does not automatically provide a safe harbor for districts that do have more than a majority share of their areas...
within their Equal-Land-Area Circles. Such districts may still be perceived as impermissible and may need to comply with additional, more complex rules, to be fashioned over time. We thus leave open the question of what set of rules may eventually provide a safe harbor for those engaged in redistricting. We only provide a simple first test for identifying impermissible gerrymanders that can be applied now, but refrain from providing a test for identifying permissible ones. That said, we believe that we have opened up a new frontier for the search for that elusive standard, that simple yet robust ‘safe harbor’ standard—the one that can identify and exclude all election district shapes that have been aggressively gerrymandered.

I The Legal Foundation for the Perception of Voter Exchange as a Constitutional Harm

1. Gerrymandering and the Supreme Court

Members of the U.S. House of Representatives are elected to office every two years\(^8\) from 435 single-member districts. While all fifty states are entitled to at least one district, the number of districts apportioned to each state is proportional to its population. State legislatures are responsible for creating redistricting maps\(^9\) that apportion equal numbers of

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\(^8\) U.S. Constitution, Article I, Section 2.

\(^9\) U.S. Constitution, Article I, Section 4. See also Chapman v. Meier, 420 U.S. 27 (1975) (Blackmun, J. opinion of the Court): “[R]eapportionment is primarily the duty and responsibility of the State through its legislature or other body, rather than of a federal court.”
people to each district, ensuring that districts are contiguous and ‘reasonably compact’, that they generally follow municipal and county boundaries, and that they encompass communities of interest. A state legislature is required to approve new redistricting plans after a decennial census determines that the number of districts in the state was altered or that the distribution of population in the state has changed.

The U.S. Supreme Court has been called upon to adjudicate disputes that have arisen from challenges to state redistricting plans on the grounds that they are excessively gerrymandered for political gain, where gerrymandering has been typically defined in general terms as “the deliberate and arbitrary distortion of district boundaries and populations for partisan or personal political purposes.”

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10 In Karcher v. Daggett, the U.S. Supreme Court reaffirmed its position that no level of population inequality among congressional districts that is too small. As Justice Brennan wrote in his majority opinion: “We thus reaffirm that there are no de minimis population variations, which could practically be avoided, but which nonetheless meet the standard of Art. I, Sec. 2, without justification.” Karcher v. Daggett, 462 U.S. 734 (1983) (Brennan, J., opinion of the Court).

11 These requirements are usually referred to as ‘traditional districting principles’.

12 The Court has not ruled out the introduction of a new redistricting plan (even in mid-decade) to reflect a political realignment: “With respect to a mid-decade redistricting to change districts drawn earlier in conformance with a decennial census, the Constitution and Congress state no explicit prohibition.” League v. Perry, 548 U.S. 17 (2006) (Kennedy, J., opinion of the Court)

There is no question that gerrymandering is pervasive, and there is good reason to believe that it is now more pervasive than ever. In this essay, we present solid empirical evidence\(^\text{14}\) that gerrymandering—be it racial, partisan, or bi-partisan—has lead to a significant decrease in congressional district compactness over time, along the entire spectrum of available measures, giving us ample reason to suspect that it is not “a self-limiting enterprise” as suggested by Justice O’Connor.\(^\text{15}\)

In 1986, the Supreme Court, in Davis v. Bandemer\(^\text{16}\), granted judges the power and the duty to control excessive political gerrymandering. But the Court—while overturning “redistricting legislation that is so extremely irregular on its face” in 1992\(^\text{17}\)—has so far declined to provide lower courts with a clear standard that can help judges determine when a shape of a district is so irregular as to be considered unconstitutional. Gerrymandering thus continues unabated and seemingly unstoppable, leading to less and less compact district shapes over time, with no remedy in sight.

As it stands, the issue before the Court is no longer whether or not there is a need, or a Constitutional basis, for restraining gerrymandering. The issue is whether a standard for


\(^{16}\) In Davis v. Bandemer, the Court ruled that “political gerrymandering cases are properly justiciable under the Equal Protection Clause”. Davis v. Bandemer, 478 U.S. 143 (1986) (O’Connor, J., concurring).

restraining aggressive gerrymandering can be articulated, a standard that is firmly 
grounded in the Constitutional harms that gerrymandering perpetrates. In 2004, in *Vieth v. Jubelirer*, Justice Scalia, representing a plurality of Justices, has called for overturning *Bandemer*, on the grounds that claims of political gerrymandering are “non justiciable 
because no judicially discernible and manageable standards for adjudicating such claims 
exist”. Justice Kennedy dissented on the grounds that he did not want to “foreclose all 
possibility of judicial relief if some limited and precise rationale were found”. So far, 22 
years after *Bandemer*, no such standard agreed upon by the majority of Justices.

There is already concern among the Justices that the failure of the Court to restrain 
aggressive political gerrymandering is a “failure of judicial will to condemn even the most 
blatant violations of a state legislature’s fundamental duty to govern impartially”. And 
there is a realization that “[a]bsence sure guidance, the results from one gerrymandering 
case to the next would likely be disparate and inconsistent.” These concerns call for an 
open debate among scientists and legal scholars on the appropriate standard for restraining 
gerrymandering, a debate that must now continue in earnest before the window of 
opportunity still left open by a five-to-four majority of the Justices in 2004 in *Vieth v. Jubelirer*

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Thomas, J.).


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is closed and political gerrymandering is declared non justiciable by the Court. This essay, proposing a novel synthesis of the legal foundations underlying the redistricting process and the scientific measurement of compactness, is written in the hope of stimulating this debate.

2. Gerrymandering, Voter Exchange, and Non-Compactness

Gerrymandering cannot be understood simply as the “deliberate and arbitrary distortion” of election district boundaries for political gain. To begin with, if the distortion is deliberate it is not arbitrary and vice versa. There is no question that gerrymandering is deliberate, and, as we shall see, it is this fact that makes it possible to detect it and to restrain it in a meaningful way. Second, the design of a district of any shape that is not a perfect shape—that is to say, not an equilateral triangle, a square, a pentagon, a hexagon, or circle—can be said to be a distortion of some kind. The design of district shapes necessarily involves some distortion, and we should not expect district shapes to be perfectly symmetrical or easily recognizable geometric forms. But the practice of gerrymandering does not involve just any kind of distortion. It is indeed a deliberate distortion of district boundaries, but it is a very particular kind of distortion. We define gerrymandering as follows:

_Gerrymandering_ is the distortion of an election district shape from a more compact into a less compact one by exchanging voters of one party (or minority group) _living close by_ for voters of another party (or
minority group) living further away—so as to affect the outcome of elections.

Generally, as we shall see, the larger the district areas exchanged in this manner and the further away the areas brought into the district are relative to the expelled areas, the more aggressive the gerrymander, and the less compact the resulting shape of the district.

This definition of gerrymandering asserts that the underlying assumption and the starting point of any gerrymandering effort is that the composition of voters in a given district that is compact and cohesive is not satisfactory to those involved in the redistricting plan, for any of a number of reasons which need not be detailed here. Gerrymandering then sets out, in effect, to obtain a more advantageous composition of voters by expelling voters living nearby and then foraging for desirable voters at locations further away to replace the expelled voters. This exchange, as we shall see, makes the shape of the district less compact and the more aggressive the exchange the less compact the shape of the district. In other words, the shape of the district is the evidence that it has been gerrymandered, or, in Justice Stevens’s words, “[a]mong those well-settled principles is the understanding that a district’s peculiar shape might be a symptom of an illicit purpose in the line drawing process.”

Gerrymandering and voter exchange are, in fact, interchangeable terms, and aggressive voter exchange is the underlying cause—in fact, the only cause—of non-compact district shapes. This is illustrated graphically in figure 1.

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Figure 1: How Voter Exchange Creates Non-Compactness

The left-hand shape in figure 1 is that of a hypothetical circular district. It is not gerrymandered and it is perfectly compact in the sense that all locations in the district are as close as possible to each other as well as to the center of the district, and no location can be brought any closer.\textsuperscript{23} The circle is, in fact, the ideal shape against which we traditionally measure compactness. It is safe to say that almost all, if not all, the proposed measures of the compactness of a given shape in the literature compare it to a circle.\textsuperscript{24} Dictionaries typically define a compact object as one closely and firmly packed, or as having component parts closely crowded together. A maximally compact election district will have the shape

\textsuperscript{23} For a complete discussion of the circle as the most compact of shapes both in terms of having the shortest average distance to a given point (its center) and in terms of having the shortest average distance (or distance-squared) among all points, see Angel, S., J. Parent and D. L. Civco, 2008, “Ten Compactness Properties of Circles: A Unified Theoretical Foundation for the Practical Measurement of Compactness”, Papers in Regional Science, forthcoming.

\textsuperscript{24} This is true in the general geographic literature, as well as in the redistricting literature. For example, “Underlying all dispersion measures is the notion that a perfect district is a regular, simple shape, usually a circle”, Pildes and Niemi, 1993, 554.
of a circle, and—as we shall see later—ordinary people, when asked to detect compactness assign district shapes that approach the shape of a full circle higher compactness values that those they assign to district shapes that deviate from a circle.25

In the middle shape in figure 1, voters in 21% of the area closer to the center were expelled from the original district and voters in an area the same size were brought into the district from further away. The resulting district shape has exactly the same area as that of the circle, but it is now less compact. In the right-hand shape in figure 1, voters in an additional 34% of the area closer to the center were expelled from the original district and voters in an area the same size were brought into the district from further away. The new district shape still has the same area as that of the circle, but it is now even less compact. The reader can ascertain that the district shape becomes less and less compact with more and more of the area housing voters living close to the center of the original district exchanged for areas housing voters living further away.

The Justices, when discussing gerrymandering, often allude to its voter exchange aspect: “A district that reaches out to grab small and apparently isolated minority communities is not reasonably compact”26; “specific protuberances on the draconian shape that reach out to include Democrats, or fissures in it that squirm away from Republicans.”27; “incorporated

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25 See Annex II: The Appearance of District Compactness is Measurable—The Empirical Evidence.


multiple, small, *farflung pockets* of minority population”\(^{28}\); “its ‘many narrow corridors, wings, or fingers… *reach out* to enclose black voters, while excluding *nearby Hispanic residents*”\(^{29}\) (italics ours).

There is a clear underlying sense in the Justices’ statements that it should be intuitively clear to everyone—both those engaged in gerrymandering and those seeking to restrain it—that some locations are ‘close by’ and some are ‘further away’. They do raise the question: ‘close by’ or ‘further away’ from *where*? and, indeed, the detection of gerrymandering in quantifiable terms requires a more rigorous mathematical or geometrical definition of what it means for some locations to be close by and for others to be further away.

There are two different senses in which we can speak of some locations being close by or further away in an election district in simple mathematical or geometrical terms. We can speak of some locations being close to or further away from the *center* of the district, the presumed location of its electoral seat. We can also speak of some locations being close to or further away from the *entire district*.

The realization that a district has a *center*—a point of maximum proximity to the entire district—is central to understanding the true nature of gerrymandering, and to our ability to detect it and measure it. Indeed, the realization that the center of an election district is a


point of special importance goes beyond its critical role in the detection of gerrymandering. For one, because it is the most accessible location in the district, its constituents should expect—in the absence of mitigating factors—that the office of the district’s representative to Congress will be located near it, so as to be in closest proximity to them.

The intimate connection between centrality and representation has indeed been recognized since the early days of the Republic: Equal representation has been traditionally bound with equal access. As Rosemarie Zaggari notes in her essay on “the Centrality of State Capitals”:

Between 1776 and 1812, all but two of the states shifted their capitals.... The demands for removal were based not just on a desire to minimize the difficulty of travel to the capital but also on a principled awareness of what those distances implied for representation.... [as] James Madison observed in 1790, ‘In every instance where the seat of Government has been placed in an uncentral position, we have seen the people struggling to place it where it ought to be (emphasis added). Americans believed that it was a matter of right, not simply personal comfort, to have a centrally located capital. As Virginia’s removal statute noted, ‘The equal rights

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of all the said inhabitants require that such seat of government should
be as nearly central to all [as possible].”

The framers of the Constitution did suggest that the proximity of representatives to their constituents was an expected feature of single-member voting districts, because it ensured their familiarity with their constituents’ interests and circumstances. As Hamilton and Madison wrote in *Federalist Paper 56*:

> It is a sound and important principle that the representative ought to be acquainted with the interests and circumstances of his constituents…. Divide the largest state into ten or twelve districts and it will be found that there will be no peculiar interests in either, which will not be within the knowledge of the Representative of the district.”

We can also speak of a voter being ‘close by’ or ‘further away’ from all other voters in the district; from the entire district, that is, and not simply from the center of the district. The cohesion of a district, the degree to which voters form a compact spatial community, where everyone lives as close as possible to everyone else, involves focusing on how far a voter’s residence may be from all locations in the district. The Court recognized district cohesion when it determined that “a district would not be sufficiently compact if it [were]

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so spread out that there was no sense of community”. People ‘gravitate’ towards each other to create cohesive communities, and it may matter to people how far they are from the community as a whole rather than simply from its center.

Interestingly enough, the two senses in which we can speak of voters as being ‘close by’ or further way’, while seemingly different, are in fact interchangeable. In other words, in purely geometrical terms the statements ‘being close to the center of the district’ and ‘being close to the entire district’ turn out to be essentially identical. If a voter in one location is further away from the center of a given district than another voter, she is also further away from the entire district than that second voter.

From the perspective of the lay person not versed in mathematics or geometry, the fact that the two definitions are interchangeable greatly simplifies things: We can speak of being ‘close by’ or ‘further away’ from a given district simply as being at a certain distance from the center of the district. By simply measuring the straight-line distances from the residence of

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35 In purely mathematical terms, we define the center of an election district as its centroid, or center of gravity, the point that has the minimum distance-squared to all other points in the district. We define the closeness of a given point to all other points in the district as the average of its distance-squared to all these points. The mathematical proof that two points that are equidistant from the centroid of a given shape have the same average distance-squared to all other points in the shape is given in Angel, S. and Hyman, G.M., 2009, Theorems Concerning the Compactness of Circles, Geographical Analysis (forthcoming).
any given voter to the centers of all districts in her vicinity, we can determine, for example, if she is voting in the district closest to her or not. The center of a district of any given shape can usually be fingered on a map of the district simply by eyeballing it. It can be determined quickly and precisely with any of a number of computer programs that locate the centroid or center of gravity of geographic shapes.

We can take this insight one step further when we realize that any election district of any shape has a hypothetical circle associated with it. This circle will have the same land area in the state in which the district is located, and its center will be at the geographic center of the district. In fact, this circle is the “baseline for calculating what would be the proper distribution of electoral outcomes in the absence of the gerrymander” that the Court appears to be seeking.  

This baseline can also function as the hypothetical district that Justice Souter is searching for, as one of his five required elements for making a case against gerrymandering: “The plaintiff would need to present the court with a hypothetical district including his residence”.  

Surely, there can be many shapes and types of hypothetical districts, some more realistic than others, some focusing on a single district while ignoring the shape of its adjoining districts, and some focusing on the shape of a hypothetical district as part of a larger State plan. We claim that the relevant baseline—and the only one necessary—for detecting the

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severity of gerrymandering in the design of a given district is the *Equal-Land-Area Circle*. This is a circle about the geographical center of the district, containing an area of land in the State equal to that of the district (see figure 2). This hypothetical district shape ignores the shapes of adjoining districts, because in the pursuit of gerrymandering we only need to examine one district at a time. Clearly, in the absence of gerrymandering, constituents in all locations in this hypothetical circular district will be as close as possible to the center of the district as well as to each other. Under these conditions, the hypothetical district will take on the shape of a circle.\(^{38}\) We can consider this hypothetical circle to be the *natural* shape that the district will have taken in the absence of gerrymandering.

![Figure 2: The Equal-Land-Area Circle, Areas Housing Expelled Voters, Voters Still in the District, and Areas Housing Foraged Voters.](image)

\(^{38}\) The reader can easily see that the circle is the geometric shape where all points are closest to a given point—its center. Any shape that is not a circle could have parts of it moved closer to its center. When no parts can be moved any closer to the center anymore, the shape will be circle.
To sum up, the shape of any district can be said to have been created—in principle, though not necessarily in practice—from an original hypothetical district. That hypothetical district had the shape of a circle, which is the natural shape of any district in the complete absence of gerrymandering. The center of that circle was the geographic center of the district, and the land area of that circle (excluding water bodies or areas outside the State) was exactly equal to the land area of the district. The actual district can be said to have been created by a process of voter exchange, expelling voters from areas inside the original circle and replacing them with voters living in areas outside the circle. Gerrymandering can be said to have occurred when large areas housing voters in the original circle were exchanged for equivalent areas outside the circle, and when voters living very close to the center of the district were exchanged for voters living very far away. More generally, the greater the share of the area of the original circle exchanged to create the district, and the greater the distance of foraged voters to the center in comparison with the distance of the expelled voters they replaced, the more aggressive the gerrymander, and the less compact the resulting shape of the district.

This perception of gerrymandering raises an important question: Looking at a State districting map, can an individual voter determine, in no uncertain terms, whether or not she has been a victim of aggressive gerrymandering? Yes, but only with reference to a hypothetical district—like the Equal-Land-Area Circle on the right in figure 2—and, unfortunately, not simply by checking whether she still votes in the district nearest her. As we noted earlier, every district has a center and an Equal-Land-Area Circle can be drawn
about that center. An individual voter can find herself inside that circle but outside the
district in question (e.g. in the cross-hatched area in figure 2, right). She can then claim to
have been expelled from the district when it was gerrymandered, and that her expulsion
constituted a Constitutional harm, as we shall argue below. Another voter can find herself
inside the district, but outside that circle (e.g. in the hatched area in figure 2, right). That
voter can claim to have been brought into the district when it was gerrymandered, and that
her abduction constituted a Constitutional harm, as we shall argue below. The remaining
voters, those that are both inside the district and inside the circle (e.g. in the black area in
figure 2, right) cannot claim to have been victims of gerrymandering.

As a result of gerrymandering, some displaced voters may still find themselves voting
in a district nearest them, a district which may be quite compact. But that does not mean
that they have not been victims of gerrymandering. This is illustrated in the left map in
figure 3, showing a pair of Congressional districts in 2004, Illinois Districts 4 and 7, and their
associated hypothetical circles. There is no question that district 4 has been aggressively
gerrymandered. In fact, voters in 72% (!) of the area of its hypothetical circle have been
expelled from the district to create it. They were exchanged for voters that were brought in
from areas outside that natural circle. Many of the voters that were expelled from District 7

39 The center can be eyeballed. A rough approximation of that circle can be obtained simply by
drawing a circle with what appears to be the same land area about the center of the district. It
can be done very precisely with any of a number of GIS (Geographic Information System)
programs, like ArcGIS.
voted in District 4, which is relatively compact, and for many of them, District 4 is now, in fact, closer than district 7. But the fact they now vote in the district nearest them should not distract us from understanding that they were victims of the aggressive gerrymandering of district 4.

![Figure 3: Illinois Congressional Districts 4 and 7 (left) and Texas Districts 15 and 25 (right), 2004](image)

**Note:** The colors of Equal-Land-Area Circles shown correspond to the color of their respective districts. Illinois District 4 on the left and Texas District 25 on the right are in light grey.

In short, we can only detect harm to an individual from aggressive gerrymandering by examining whether or not she can claim to have been expelled from a particular district, and she can make that claim if she can show that her residence was within the hypothetical Equal-Land-Area Circle associated with that particular district. In fact, in exceptional cases a voter may claim to have been expelled from *more than one* district due to aggressive gerrymandering. This is illustrated in the map on the right of figure 3, showing a pair of Congressional districts in 2004, Texas Districts 15 and 25, and their associated hypothetical circles. There is a small area shown in black that is part of both circles, but not part of either
district. Voters in this area where expelled from voting in either district in the process of aggressive gerrymandering.

To conclude, an individual voter can claim to have suffered harm from gerrymandering if she can show that she has been expelled from voting in a particular district or that she was brought in to vote in a particular district, by reference to the hypothetical circle associated with that particular district. Her claim is not that she was not allowed to vote in the district nearest her or that she ended up voting in a district which was not compact. Her claim is, rather, that in the absence of gerrymandering she would have voted in her natural district, the district she belonged to, the district that she expected to vote in had it not been for gerrymandering. In that un-gerrymandered district, she was indeed closest to the district as a whole, as well as to the center of the district, because it was maximally compact. The fact that she was displaced to another district that is also nice and compact or that she is now closer to that new district than to her original district is no consolation at all.

What is the nature of the harm that she can claim to have suffered?

3. Voter Exchange as a First Amendment Harm

Given our definition of gerrymandering as the exchange of voters living close by—inside a hypothetical circle associated with an individual district—for voters living further away, we suggest that the most effective way of curing it may be by defining it as a Constitutional harm that infringes on voters’ First Amendment rights. Departing from earlier attempts to
posit an array of doctrinal tools to ground such harms, we follow Justice Kennedy’s suggestion in *Vieth v. Jubelirer*:

The First Amendment may be the more relevant constitutional provision in future cases that allege unconstitutional partisan gerrymandering. After all, these allegations involve the First Amendment interest in not burdening or penalizing voters because of their participation in the electoral process, their voting history, their association with a political party or their expression of political views. First Amendment concerns arise where a State enacts a law that has the purpose and effect of subjecting a group of voters or their party to disfavored treatment by reason of their views.

First Amendment rights are thus compromised if State legislatures draw district lines that expel large numbers of voters from voting in the district in which they would expect to vote, exchanging them for voters living further away. Voters who expected to vote in a particular district—in which, in the absence of aggressive gerrymandering, they would have

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been closest to other voters as well as to the center of the district, its presumed electoral seat—are thus barred from voting in that district nearest them in favor of preferred voters living further away, for no reason other than their “voting history, their association with a political party, or their expression of political views”.

A plaintiff living close by, inside the hypothetical natural district associated with a highly gerrymandered district, who was expelled from voting in the district may thus have standing in arguing before the Court that she was barred from participating in voting in the district, where she—together with other, similarly expelled, neighbors—could expect to vote. Because gerrymandering necessarily involves aggressive voter exchange, had politics played no part in the districting decision that barred her and her neighbors from voting in the district, there would have been no reason to make the district less compact, and she would have been allowed to vote there.\(^43\) Allowing for a large margin of error, the larger the areas within that hypothetical circle that were exchanged for outlying ones and the further away outlying areas were foraged to replace expelled voters from areas inside that natural circle, the stronger her case will be.

The claim that she has suffered a First Amendment harm in this instance does not necessary imply that she suffered a material loss—say a loss of a job or a contract—because

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\(^43\) A claim that a district was gerrymandered to include the home of a representative—typically an incumbent politician—living far away from the center of the district, thereby leading to the rejection of many voters living closer to the center of the district creates the same First Amendment harm.
of her voting history. It is rather that she suffered from unfair discrimination that required her to vote in a district other than her ‘natural’ district, the one in which she would expect to vote in the absence of gerrymandering. By analogy, requiring blacks to sit at the back of the bus, rather than in the first available seat they find, is not a material harm. The bus brings both those seating in the back and those seating in front to their desired destinations with equal speed and dispatch. The harm suffered is their expulsion from the front of the bus because of the color of their skin, although there is no material advantage to sitting at the front of the bus, except for a slightly shorter walk to one’s seat. Similarly, the First Amendment harm to voters expelled from their ‘natural’ district, the one in which they would expect to vote in the absence of gerrymandering, because of their voting record is not a material harm, requiring them to drive further out, say, to meet their representative at his or her office. It is the harm perpetrated by discriminating against them for no reason other than their voting record.

By purposely rejecting an individual voter from voting in their ‘natural’ district for no other reason than her voting history so as to fashion exclusive voting districts that serve this or that political objective, the State can be shown to exercise discriminatory purpose. The Court has defined discriminatory purpose as follows:

“Discriminatory purpose,” however, implies more than intent as volition or intent as awareness of consequences. It implies that the decisionmaker, in this case the state legislature, selected or reaffirmed
a particular course of action at least in part “because of,” not merely

“in spite of,” its adverse effects upon an identifiable group.\textsuperscript{44}

The ‘identifiable group’ in this case is the group of expelled voters. The ‘adverse effect’ on these voters is barring them from voting in the district they would be expected to vote in, with the expressed purpose that their votes \emph{would not be counted} in this district. The ‘discriminatory purpose’ is in making sure that their votes not be counted when the votes in this district are tallied solely because of their voting history, and for no other reason.

This is not to suggest that \textit{a single voter} that was rejected from voting in the district in which she would be expected to vote and replaced by a voter living further away can successfully claim in court that she suffered a First Amendment harm because of her voting record. The Court recognizes that redistricting needs to fulfill a number of requirements—especially the requirements that districts be contiguous, that they have equal populations, that district boundaries follow existing administrative boundaries, and in some cases that a minority form a majority in the district or that an incumbent be reelected. It also recognizes that a State legislature can easily demonstrate that, given these requirements, there exists no redistricting plan that ensures that \textit{everyone} votes for his or her natural district. And it is aware that any quantitative standard it chooses to apply may be subject to definitional and measurement errors. For those reasons, any standard chosen by the Court to restrain aggressive gerrymandering must, of necessity, have \textit{a wide margin or error},

\textsuperscript{44} Personnel Administrator MA v. Feeney, 442 U.S. 279 (1979).
allowing a State legislature sufficient—while not excessive—latitude in the preparation of redistricting plans.

A plaintiff claiming to have been prevented from voting in the natural district in which she would be expected to vote must therefore demonstrate that she was one among many voters in a large area of that hypothetical district—say, a majority share of the area—that were expelled from it because of their voting history. If the Court—allowing for a wide margin or error—agreed on a standard for determining, in no uncertain terms, how large a share of that hypothetical district area is too large then the plaintiff would know if and when she has grounds to sue the State legislature. If voters like her from too large a share of the area of the hypothetical district were expelled then she can claim with some certainty that her First Amendment rights were infringed.

Similarly, a voter that was foraged from far outside the hypothetical natural district of a highly gerrymandered district and brought into the district to vote there can justifiably claim that she was brought in for no other reason than her voting history. Again, if voters like her from too large a share of the district area were brought into the district so as to affect election results there she can claim with some certainty that her First Amendment rights were infringed.

4. Voter Exchange as a Fourteenth Amendment Harm

In *Davis v. Bandemer*, the Court ruled that “political gerrymandering cases are properly justiciable under the Equal Protection Clause”\(^{45}\) of the Fourteenth Amendment. Justice

Stevens, in *Vieth v. Jubelirer*, addresses voter exchange directly as perpetrating an Equal Protection harm. He insists that

> [P]olitical affiliation is not an appropriate standard for excluding voters from a congressional district.... Under my analysis, if no neutral criterion can be identified to justify the lines drawn, and if the only possible explanation for a district’s bizarre shape is a naked desire to increase partisan strength, then no rational basis exists to save the district from an equal protection challenge.\(^\text{46}\)

By creating election districts in which voters that live further away are included in the district while voters who live closer by are excluded from it is an indication that the shape of such election districts, in and of itself, does not respect the Equal Protection principle—some voters are clearly more desirable than others. In the words of Judge Ward, “extreme partisan gerrymandering leads to a system in which the representatives choose their constituents, rather than vice-versa.”\(^\text{47}\) By allowing elected officials to win elections by choosing who will vote for them and who will not in a deliberate and unrestrained fashion, ‘the original promise of the Court’s intervention into the political process as securing “the free and uncorrupted choice of those who have the right to take part in that choice”’\(^\text{48}\) is frustrated.

\(^{46}\) *Vieth v. Jubelirer*, 541 U.S. 339 (Stevens, J., dissenting).


As we noted earlier, in a district that is not gerrymandered, most voters would expect to live within the hypothetical circle associated with their district. In geometrical terms, the great bulk of the area of every district would be expected to be within the hypothetical circle associated with that district. Still, in both the 1996 and the 2004 Congressional elections, for example, some 10% of the 435 election districts had less than half their areas within their associated hypothetical circles, leading us to suspect that large numbers of voters living close in these districts were expelled and replaced by voters living further away.

Preventing some citizens from voting in the district nearest to attain basic redistricting objectives them may not be, by itself, a Constitutional harm. Preferring voters who live further away is, because it flies in the face of the Constitutional guarantee of Equal Protection. As Issacharoff and Karlan note:

If ‘[a]t the heart of the Constitution’s guarantee of equal protection lies the simple command that the Government must treat citizens as individuals,’ then redistricting stabs at the heart of the Fourteenth Amendment every time.49

The interpretation of gerrymandering as aggressive voter exchange is not meant to imply that voting in the district nearest you is a Constitutional right or that everyone is entitled to vote for the representative nearest them. But it does indeed suggest that most

voters could *legitimately expect* to vote in the district nearest them, unless there is a good explanation of why they should be prevented from doing so.

Suggesting that it does not matter in which district one gets to vote, and that no harm was perpetrated as long as one exercises the right to vote in a given election does not really address either the First Amendment or the Fourteenth Amendment harms discussed here. The issue at hand is that it is of key importance *in which district* one gets to vote, not whether one has participated in statewide elections. This is made patently clear in Justice Scalia’s opinion in *Vieth v. Jubelirer*. Justice Scalia, insisting that winning a majority of votes in statewide elections is irrelevant in examining which party won a specific district contest, provides the following quote, a quote that clearly establishes his opinion on the importance of voting *in a particular district* for a party competing for a *specific* electoral seat:

> [T]here are `separate elections between separate candidates in separate districts, and that it all there is. If districts change, the candidates change, their strengths and weaknesses change, their campaigns change, their ability to raise money changes—everything changes. Political parties do not compete for the highest statewide vote totals or the highest mean district vote percentages: they compete for specific seats."

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The authors propose that the Court re-examine its objection to placing restraints on gerrymandering of any kind, and recognize that non-compactness—when it discriminates against large numbers of nearby voters in favor of others further away—creates a Constitutional harm because it fails to respect the Equal Protection clause. Voters’ choice is indeed corrupted when they are prevented from voting for the representative nearest them for no obvious reason (other than their voting preferences), and it is corrupted precisely by drawing district shapes that are not compact.

5. The Appearance of Non-Compactness as an Expressive Harm

A third Constitutional harm from gerrymandering that is articulated in the literature is not a harm to particular individuals or groups, but a systemic harm, a harm to the political system as a whole. There is good cause for worry, in the words of Judge Ward quoted earlier, that “extreme partisan gerrymandering leads to a system (emphasis added) in which the representatives choose their constituents, rather than vice-versa.” The Court has recognized the harm that excessive gerrymandering inflicts on the political system, and its own duty to restrain it, but has so far chosen not to intervene. This is made plain in Justice Kennedy’s opinion in Vieth v. Jubelirer:

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51 The term ‘Expressive Harm’ was coined by Pildes and Niemi, 1993, 485.

Here, one has the sense that legislative restraint was abandoned. That should not be thought to serve the interests of our political order. Nor should it be thought to serve our interest in demonstrating to the world how democracy works. Whether spoken with concern or pride, it is unfortunate that our legislators have reached the point of declaring that, when it comes to reapportionment, “We are in the business of rigging elections”\(^53\).…..

Still, the Court’s own responsibilities require that we refrain from intervention in this instance. The failings of the many proposed standards for measuring the burden a gerrymander imposes on representational rights make our intervention improper.\(^54\)

The Court has recognized that the legitimacy of democratic elections may be compromised if voter confidence in the fairness of these institutions and in the ability of the Constitution to protect them is diminished by the persistent appearance of district shapes that are not compact. As Justice O’Connor noted in writing the majority opinion in Shaw v. Reno, “[w]e believe that reapportionment is one area in which appearances do matter.”\(^55\)

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The Court has frequently voiced its displeasure with the *appearance* of non-compact district shapes. The adjectives that justices have used to characterize them include: “oddly shaped”\textsuperscript{56}; “malapportioned”\textsuperscript{57}; “irregular”\textsuperscript{58}; “highly irregular”\textsuperscript{59} and “dramatically irregular”\textsuperscript{60}; “not a model of aesthetic tidiness”\textsuperscript{61}; “[a] tortured shape”\textsuperscript{62}; “bizarre”\textsuperscript{63}; “misshapen”\textsuperscript{64}; “a Rorschach ink-blot test”\textsuperscript{65}; “uncouth”\textsuperscript{66}; “outlandish”\textsuperscript{67}; and “grotesque”\textsuperscript{68}.

“[I]f you drove down the interstate with both car doors open, you’d kill most of the people}


\textsuperscript{65} Shaw v. Reno, 509 U.S. 635 (O’Connor, J., opinion of the Court).


in the district"; "it resembles ‘a sacred Mayan bird…. Spindly legs reach South... while the plumed head rises northward… In the western extremity of the district, an open beak appears to be searching for worms…. Here and there, ruffled feathers jut out at odd angles"; or “a bug splattered on a windshield”.

Unfortunately, none of these florid descriptions translates easily into a criterion that can be demonstrably grounded in a Constitutional harm to individual plaintiffs. We agree with Issacharoff and Karlan that “no claim of systemic harm could possibly be advanced from the vantage point of one district,” and that “the Court turned all claims—even claims that were fundamentally about political structure—into claims of individual rights.” We do believe, however, that the systemic failure brought about by excessive gerrymandering can only be corrected by continuing to focus on individual harms, and more specifically on First Amendment and Fourteenth Amendment harms.

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In exploring the possibility for advancing First Amendment and Fourteenth Amendment claims to challenge State redistricting plans, we follow the advice of Justice Souter, focusing our attention on the “suspect characteristics of individual districts instead of state-wide patterns”.74 We understand and accept that gerrymandering can only be restrained one district at a time, and we believe that the accumulation of district shapes that meet a required voter exchange standard will gradually and inexorably lead to the desired restraint of the overall systemic harm. In short, we follow the Court’s lead in searching for the harm from gerrymandering in the realm of individual rights in individual districts, while seeking to ensure that the criterion adopted for restraining it will prevent further systemic harm.

We thus posit the conjecture that the appearance of ‘bizarre’ district shapes is the direct result of aggressive voter exchange. This is a conjecture that can be proved or disproved by looking at the empirical evidence and asking a simple question: will a standard that places restrictions on voter exchange eliminate all, or most, district shapes that appear to be non-compact? In what follows we hope to convince the reader that this is indeed the case. We will attempt to show that curing the First Amendment and Fourteenth Amendment harms that results from excessive voter exchange will also cure the systemic harm to our democratic system that results from the appearance of non-compact and ‘bizarre’ district shapes. More specifically, we will demonstrate that—in the districting plans for the Congressional elections of 1996 and 2004—a standard that places a reasonable numerical

limit on voter exchange would have eliminated most, though not all, of the district shapes that were judged to be highly non-compact by independent observers.

Indeed, from the perspective of the authors, the problem thus far has been that the proposed quantitative metrics to measure district compactness have been found seriously wanting and far from useful to the Court in its efforts to restrain gerrymandering. In fact, it may be safe to say that compactness metrics—like the perimeter-area ratio, to take one example—have been and still are largely irrelevant in helping decide the redistricting cases taken up by the Supreme Court, or by any other court for that matter. They are largely irrelevant because they address the appearance of district shapes rather than focusing on the Constitutional harm to individual voters. They are also likely to remain irrelevant unless a number of their fundamental weaknesses are addressed. And in the absence of a reliable and relevant way of detecting, measuring, and restraining district compactness it is unlikely that gerrymandering can ever be restrained.

The standard we propose in this essay will regulate the underlying cause of ‘bizarre’-looking districts, rather than seeking to restrain their outward appearance directly. The appearance of non-compact and ‘bizarre’-looking districts will necessarily be restrained by this standard as well, though indirectly, gradually reducing the systemic harm from excessive gerrymandering to the legitimacy of democratic elections. And it will send a clear message both to State legislatures and to the electorate that there are Constitutional limits to the extent to which political interests can subvert and corrupt the electoral process. We have taken on this challenge because, like many others, we do understand that
[u]nless courts move in the direction of specific compactness constraints, the search for reasonable configurations of districts will become a distinct liability for territorially based electoral systems.75

II The Appropriate Measurement of District Compactness

1. Selecting Relevant Measures of District Compactness

In this essay, we propose a way of perceiving and measuring district compactness that may overcome four critical weaknesses besetting existing district compactness metrics. First, as we suggested in the previous section, they typically measure irrelevant geometric properties of shapes, properties that are not related to the underlying motive for making district shapes more or less compact, namely voter exchange. Second, they attempt to measure the supposed appearance of district shapes without insisting that the selected measures—to the maximum possible extent—actually correlate with people’s intuitive perceptions of compactness. Third, they focus on the appearance of individual district shapes as if they were on a blank background, not taking into account local geographical constraints such bodies of water and state boundaries. And fourth, they are not able to identify and consistently reject all, or most, district shapes that are perceived as the most non-compact and the most aggressively gerrymandered by most observers.

In the absence of a reliable and relevant way of detecting, measuring, and restraining district compactness it is unlikely that gerrymandering can ever be restrained. Considering that much has been written on this subject, and that most writing has failed to provide the answers that the Court is seeking, we propose to shed new light on the measurement of compactness. We propose to examine it in a novel manner that may make it more useful to the Justices in their search for an appropriate standard to restrain political gerrymandering.

In this section we address the issue of relevance, and in the following three sections we address the three remaining issues introduced earlier. There is indeed a substantial literature devoted to the measurement of the compactness of geographic shapes—such as election districts—and many measures have been proposed and debated over the past two centuries.\(^76\) One serious problem that has emerged from the proliferation of measures of

compactness is “confusion concerning the appropriate use of the various measures that have been developed”.77 This is a cause for worry because

failure to distinguish among aspects of shape opens one to the risks of falling into mathematical malapropisms78, in which measures designed to gauge one aspect are applied to another aspect. The results are predictably bad.79

How do we determine which one of two abstract shapes is more compact? Do we look for relative roundness as against elongation? Do we look for relative fullness as against perforation? Do we look for a relatively smooth as against a relatively serrated perimeter? Do we look for a relatively short perimeter? Do we look for a relatively substantial core? Do we look for points that are furthest away from each other? The answer to these questions is unfortunately ‘it depends’: it depends on the object or objects that the shape represents. Are we speaking about the cross section of a pipe? Are we speaking about the shape of a star or a planet? Are we speaking about strategic depth from the frontier? Are we speaking of accessibility in a city or about a defensive wall surrounding it? Or are we speaking of the distribution of voters in a Congressional election district?

77 MacEachern, 1985, 66.

78 Malapropism is typically defined as confusing a word for another one with a similar sound, often to humorous effect.

A compactness measure designed to capture strategic depth may not be able to capture accessibility. One designed to measure perimeter versus area in pipes and city walls may not be able to detect voter exchange. One designed to measure the existence of a substantial core may not be able to capture smoothness. It should therefore come as no surprise to the reader that numerous proposals have been advanced for using the wrong compactness property of a given shape to capture a completely different property: witness the continued reliance on various perimeter-area relationships to gauge the appearance of district shapes\textsuperscript{80}, despite their total irrelevance for detecting compactness. The two shapes in figure 3, for example, have the same perimeter-area relationship,\textsuperscript{81} but observers all agree that the one on the left is more compact, in the dictionary sense of packed closer together, than the one on the right. As for voter exchange, the figure on the left has a serrated boundary suggestive of block-by-block voter exchange at the micro level. The one on the right is long and thin, though smooth, suggestive of massive voter exchange at the macro level: rejecting many


\textsuperscript{81} Measured, in this case, as the ratio of their areas to the squares of their perimeters, a commonly used measure of this relationship.
nearby voters and replacing them with voters living further away. All that tells us is that
the perimeter-area relationship can detect neither smoothness nor what is commonly
understood by compactness, unless we first refine it and then supplement it with other
measures that can better differentiate between these key aspects of shape.

![Figure 3: Two Shapes with the Same Perimeter-Area Relationship](image)

To the best of our knowledge, no work on the subject of measuring aspects of shape
suggests a methodology that associates specific measures of compactness with the
motivation for measuring them and with the particular problems they seek to address. In
designing a cross section for a water pipe, for example, we may opt for a shape that has a
minimal perimeter for a given area, so as to minimize the amount of material used to make
the pipe. Surely the circle is the most compact of shapes in the sense of having the shortest
perimeter for a given area. But there is no reason why the relationship between the
perimeter and the area of an election district would matter in the least.

Non-Compactness As Voter Exchange

In a separate article, the authors introduced a unified theoretical foundation for the study of geographical shape compactness, integrating a select number of both old and new measures into a comprehensive framework for understanding the distinct properties of shape compactness that need to be measured, and for measuring these properties using the latest GIS (Geographical Information System) software. Before presenting a summary review of this theoretical framework here, we should note that calculations of numerical scores that characterize geographical shapes have now become much simpler with the advent of GIS software. As Justice Kennedy notes:

[T]hese new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties.

Indeed, we claim that our proposed standard for measuring district compactness responds directly to Justice Kennedy’s hope and expectation. We have made substantial use of GIS software to calculate and test no less than a dozen distinct compactness metrics for all U.S. election districts in ways that were not possible only a few years ago. In fact, since the use of GIS software is now as ubiquitous and universal as it is reliable, we consider it appropriate for the Court to adopt a compactness standard that relies on GIS software, both to derive it and to ascertain that particular district shapes meet it. If GIS software can

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be relied upon in cadastres, in the determination of administrative boundaries, and in property registration and taxation, it can also be relied upon to ensure the integrity of elections.

The proposed theoretical foundation for the study of shape compactness rests on two simple observations: First, that the circle is the most compact of shapes; and second, that there are distinct geometrical properties of the circle that make it the most compact of shapes. These distinct compactness properties of shapes, taken together, provide a new perspective on what constitutes shape compactness in the broadest sense. They make it possible to select the particular property that needs to be measured to detect compactness or its lack thereof before proceeding to measure it. This is important, because each property typically corresponds to different forces acting on the shape or to different aims that the shape may seek to attain. In other words, before attempting to measure compactness of a given shape, we should ask ourselves what is the underlying function that the shape appears to perform or needs to perform, or what are the underlying forces acting on that shape to give it form. With that in mind, we can confidently proceed to select the correct compactness property that needs to be measured—the one that best corresponds to the function that the shape seeks to perform. The discovery of relations between form and function can be traced back to Aristotle, but is best captured in a quote by Louis Sullivan, one of the pioneers of Modern architecture:

It is the pervading law of all things organic and inorganic,

Of all things physical and metaphysical,
Of all things human and all things super-human,

Of all true manifestations of the head,

Of the heart, of the soul,

That the life is recognizable in its expression,

That form ever follows function. This is the law.85

Each compactness property of a given shape needs to be understood in terms of its underlying function or purpose. If no function or purpose can be discerned or stipulated, then it should not matter at all whether the shape is compact or not. To investigate the compactness of a given shape we must, therefore, first determine or stipulate the function that the shape needs to perform or identify the forces acting on the shape, and only then focus on the particular geometric property that these functions or forces seeks to affect so as to make the shape more compact or less compact. There is no sense, in other words, in measuring shape attributes that are irrelevant to a specific investigation of compactness that has some purpose in mind. To quote Pildes and Niemi:

In the absence of a clear conceptual understanding of what dimensions of district appearance are relevant, the basis for judging districts will be unclear.86


86 Pildes and Niemi, 1993, 541.
The reader should note that most of the senses in which the circle is the most compact of shapes—and on which most of the proposed metrics in the literature are based—are irrelevant to detecting district compactness, because they do not relate it to the motivations for creating compact or non-compact districts. Only three properties of the circle are indeed relevant in the matter of district compactness: its Area Exchange property, its Proximity property, and its Cohesion property. We elaborate on these three properties below.

The Area Exchange property of any given shape is the share of the area of the shape in an Equal-Area Circle about the center of the shape. It focuses directly on the voter exchange aspect of gerrymandering, a focus that is crucial for understanding the underlying meaning of district compactness or its lack thereof. Non-compact district forms have a purpose and a function. And that function—exchanging voters living close by for voters living further away for political gain—is what gives them their form. In other words, the misshapen form of an aggressively gerrymandered district is a direct result of its function, and it cannot be understood, explained, or measured without reference to this function. In contrast, as we explained earlier, a district shape resembling a circle will have maximum compactness in terms of area exchange. It will exactly overlap an Equal-Area Circle about its center because no exchange of voters has taken place.

The Proximity property of any given shape focuses on the extent to which the shape is packed around its center. For the purpose of measuring district compactness, it measures

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87 The Equal-Land-Area Circle, which is a refinement of the Equal-Area Circle will be introduced in Section IX below.
the degree to which all district voters are close to the geographic center of their district, the presumed location of its electoral seat. The further district voters are from the center of the district, the less compact and the more gerrymandered a district is. Conversely, the closer district voters are to the center of their district, the more that district will resemble a circle.\footnote{Circus tents, for example, have a circular perimeter precisely to embody this compactness property of circles, seeking to have everybody as close to the action as possible. By analogy, in the absence of gerrymandering, voters would expect to be as close as practicable to the geographic center of their district, the presumed location of its electoral seat. Under these ideal conditions, the shape of their electoral district will be a circle.}

Circus tents, for example, have a circular perimeter precisely to embody this compactness property of circles, seeking to have everybody as close to the action as possible. By analogy, in the absence of gerrymandering, voters would expect to be as close as practicable to the geographic center of their district, the presumed location of its electoral seat. Under these ideal conditions, the shape of their electoral district will be a circle.

The \textit{Cohesion} property of any given shape focuses on the average distance among all its points, rather than on the average distance to a given point. It is a measure of how close all points in the shape are \textit{to each other}. For the purpose of measuring district compactness, it measures the degree to which voters form a compact spatial community, where everyone lives as close as possible to everyone else. The Court has recognized cohesion compactness when it determined that “a district would not be sufficiently compact if it [were] so spread out that there was no sense of community”.\footnote{People ‘gravitate’ \textit{towards each other} to create cohesive communities, and the closer members of a community are to each other, the more this community will resemble a circle. Stars, for example, have a spherical shape because \begin{itemize}
\item An similar index that also measure the closeness of a district shape to its center is the \textit{Moment of Inertia} Index.
\end{itemize}
the particles of plasma of which they are made gravitate towards each other, seeking to get as close to each other as possible. In a spherical star, the average distance\textsuperscript{90} between all particles is at a minimum, and moving any particle to a different position relative to others will only increase this average distance. By analogy, the overall shape of a cohesive community—where everyone is as close as practicable to everyone else, will take the shape of a circle. And if that community was large enough or small enough to form an election district, the shape of that district will also resemble a circle.

We can conclude, therefore, that all three compactness properties of shapes—the Area Exchange property, the Proximity property, and the Cohesion property—are highly relevant measures of district compactness. In general, we should also expect a district shape that is compact in terms of Area Exchange to exhibit high Proximity and high Cohesion. The compactness indices that measure the area exchange, proximity, and cohesion of election district shapes\textsuperscript{91} were found to be very highly correlated with each other. We found the correlations among them, both for the 435 election districts for the 108th Congress in 1996 and for the 110th Congress in 2004 to be all greater than 0.95. And because all three relevant measures were found to be highly correlated with each other, we do not need all three to detect gerrymandering. One will ordinarily suffice. We therefore propose to focus on one of them—Area Exchange—as the single, and in our view most relevant measure of district compactness.

\textsuperscript{90} Or the average distance-squared.

\textsuperscript{91} The exact definitions of these indices are given in Annex I.
We look again at the Equal-Area Circle, the circle about the geographic center of the district with the same area as that of the district in question. The quantity of voter exchange can be detected by measuring the amount of overlap between that circle and the district shape. The ratio of the area of overlap to the area of the district—referred to here as the Area Exchange Index—is, in fact, the relevant measure of compactness here, because it is a proxy measure for the amount of voter exchange: in the absence of data on the distribution of the population over space, the smaller that share the greater the quantity of voter exchange. Figure 4 shows the shapes of three U.S. Congressional districts in 2004. District 25 in Texas, for example, was found to have one of the lowest scores on the Area Exchange Index—0.3 (see figure 4, left). District 13 in Ohio (figure 4, center) had exactly 50% of its shape within its corresponding Equal-Area Circle, and therefore a score of 0.5 on the Area Exchange Index. District 4 in Arizona (see figure 4, right) had the highest score on the index—0.9.

92 The use of the Area Exchange Index, as defined here, as a measure of compactness is described in Whittington, G., K.S.O. Beavon and A.S. Mabin, 1972, “Compactness of Shape: Review, Theory, and Application.” Environmental Studies, Occasional Paper No. 7, Department of Geography and Environmental Studies, University of Witwatersand, Johannesburg, 12-13.

93 Most of the districts with even lower scores on the Exchange Index were split apart by bodies of water. The Exchange index, in its present form, does not account for separation by bodies of water. We take up this issue in the following section, where we introduce the landscape constraints.
Figure 4 demonstrates the efficacy of the Equal-Area Circle about the center of the district as a helpful aid in detecting compactness. The larger the area of the district inside that circle, the more compact the district. More importantly, the larger the area of the district inside that circle, the lesser the area that was exchanged to create the district, and the less gerrymandered the district.

Figure 4: Area Exchange Compactness in U.S. Congressional Districts in 2004; Left—Texas Dist. 25 (0.34, Low); Middle—Ohio Dist. 13 (0.49); and Right—Arizona Dist. 4 (0.91, High).

2. Selecting Measures that Correlate with People’s Perceptions of Compactness

We next address the question of measuring the appearance of compactness of election district shapes. In deliberations concerning district compactness, to quote Justice O’Connor again, “appearances do matter”.94 One of the functions of district shapes must therefore be

to appear to be compact,\textsuperscript{95} because, as argued earlier, the appearance of compactness, in and of itself, has an intrinsic value: the appearance of non-compact districts creates expressive harms and damages our confidence in public institutions and in democratic elections. As we shall see, all three properties—area exchange, proximity, and cohesion—provide excellent measures of the appearance of the compactness of shapes on a blank background.

In Annex II, the authors report on a pilot survey that examined the degree to which there is common agreement among individuals about the perceived relative compactness of district shapes.\textsuperscript{96} In this survey, we sought to measure the appearance of district compactness by giving 81 disinterested volunteers a page with nine district shapes drawn on it and asking them to rank them in order of decreasing compactness (see figure 5).

\textsuperscript{95} It is not exactly clear what that means in actual practice. Do we mean that the district shape should be assessed for compactness as a blot on a blank background? Can the image include topographic features and administrative boundaries? Should it appear to be compact to the non-trained eye or to someone more experienced in detecting compactness? Can visual aids be used to help in assessing the appearance of compactness? Can verbal and written instructions be given to alert people to what to look for when judging the appearance of district shapes? The answers to these questions are likely to affect people’s assessment of the appearance of compactness to a significant degree. Pildes and Niemi (1993, 536) suggest that “only perceptions that are properly informed, for example, and generated under normatively appropriate conditions could plausibly be relevant”.

\textsuperscript{96} There is no reason to suspect that community standards would vary when it comes to detecting compactness.
Our first important finding was that people share their perceptions about the appearance of compactness to a very high degree. Indeed, we can say with 99% confidence that the average compactness ranking people in the general population will assign to any shape (in this survey) will not vary by more than half a rank in any direction. We then calculated the correlations between the average perceived compactness rankings for each of the nine districts by survey participants and the scores on twelve selected compactness indices for each of these districts. We found that the scores on the three indices based on area-exchange, proximity, and cohesion had very high correlations—all in excess of 0.91—with people’s perceptions of the appearance of compactness. The Cohesion Index has correlation of 0.91 with the survey results, the Proximity Index had a correlation of 0.91, and
the Area Exchange Index had a correlation of 0.93. In other words, these three indices were able to predict perceived compactness, an excellent measure of the appearance of election districts, with a very high degree of accuracy. And they had higher correlations with perceived compactness than several other indices proposed in the redistricting literature. Again, because all three measures were found to be highly correlated with people’s perceptions of the appearance of compactness, as well as with each other, we do not need all three to detect gerrymandering. One will ordinarily suffice. As noted earlier, we propose to focus on one of them, Area Exchange, albeit in a slightly modified form, as the single—and in our view the simplest and the most appropriate—measure for detecting the appearance of compactness in district shapes.

3. Selecting Measures that Take into Account Local Geographical constraints

Hawaii’s 2nd Congressional district, to take one example, includes all the Hawaiian Islands except of a small area around Honolulu. The islands are separated by vast expanses

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97 The score on the Moment-of-Inertia Index also had a high correlation, 0.89, with the survey average, but it is not discussed here further because it is less relevant to the understanding of district compactness. The moment of inertia of a given shape is not related in any conceivable way to the distribution of voters in election districts.

98 The Perimeter and Dispersion Indices proposed by Pildes and Niemi, 1993, 554-555, for example, had much lower correlations, 0.52 and 0.59 respectively, with the average perceived compactness in our survey. The Fractal Index proposed by Wildgren, 1995, 111-112, had a correlation of 0.59.
of the Pacific Ocean, making the district non-compact by any measure. But this non-compactness has really nothing to do with gerrymandering, and a lot to do with the natural geography of the state. And the same is true for a number of other districts that border on water bodies, be they oceans, lakes, or rivers. Schwartzberg, one of the early students of district compactness, noted that “[c]urvatures and extensions created by natural boundaries such as meandering streams may make it exceedingly difficult to determine the compactness of a district”. He also observed that “[o]f the 435 existing congressional districts the compactness of only a dozen or so appear to me to be affected to a significant degree by the shape of the state”.99 There is no question that bodies of water and state boundaries do affect the shapes of districts, rendering them non-compact even in the complete absence of gerrymandering.

This limits the degree to which compactness measures of any kind can detect gerrymandering. Indeed, all three indices introduced earlier—the Area Exchange Index, the Proximity Index, and the Cohesion Index—identify Hawaii’s district 2 as non-compact, even though we know that it was not gerrymandered. If we are to propose a quantitative compactness standard for detecting aggressive gerrymandering, then this standard must allow districts that are made non-compact by water bodies or state boundaries to pass while flagging districts that have actually been made non-compact by aggressive gerrymandering.

This can be accomplished by shifting our focus from the Equal-Area Circle as the hypothetical district against which we measure compactness to the Equal-Land-Area Circle.

The Equal-Land-Area Circle is also a circle about the geographic center of the district with the same area as that of the district, but with a difference: the radius of the circle is adjusted so that the circle contains *land area in the State* equal to the land area of the district, excluding water bodies and areas in neighboring states. Again, the smaller the area of the district outside that circle, the less suspect it is of aggressive voter exchange. The relevant metric for the quantity of voter exchange using this circle is the **Voter Exchange Index**. It measures the ratio of the area of overlap that the district shares with the Equal-Land-Area Circle:

The **Voter Exchange Index** is the share of the area in the Equal-Land-Area Circle that houses voters that still vote in the district.

The advantage of this index over the Exchange index introduced in the previous section is that it takes into account geographical constraints. Indeed, this index does not identify districts that border on water bodies or on a neighboring state as less compact and more aggressively gerrymandered than they really are. Considering that the detection of the appearance of compactness can be improved with training as well as with visual aids\(^{100}\), so that observers take into account geographical features and state boundaries, we can conclude that the Voter Exchange Index has the potential for detecting the expressive harms that are caused by the appearance of non-compact ‘bizarre’ districts. Observers can learn to

\(^{100}\) This may entail looking at district shapes on simplified maps, such as those in figure 6, showing bodies of water and areas outside the state, as well as the Equal-Area and the Equal-Land-Area Circles, instead of looking at silhouettes of shapes on a blank background.
take into account water bodies and state boundaries before identifying a particular district shape as having been aggressively gerrymandered.

Figure 6: Voter Exchange Compactness in U.S. Congressional Districts in 2004: Left—Ohio District 9 (0.39) and Right—Florida District 8 (0.38).

Note: The area-exchange circle on the left is shown in dotted line. It contains a smaller share of the area of the district, and therefore the two districts have different scores (0.30 and 0.38 respectively) on the Area-Exchange Index.

Figure 6 shows two U.S. Congressional districts in 2004, Ohio district 9 (left) and Florida district 8 (right). Both had very similar scores on the Voter Exchange Index, 0.39 and 0.38 respectively. But they had quite different scores on the Area Exchange Index defined earlier, 0.30 and 0.38 respectively. Looking at Ohio district 9 without knowing that it is located on Lake Erie, an unbiased observer would assign it a lower compactness score than to Florida district 7. It appears to be less compact. But if we were to properly account for the fact that the two parts of Ohio district 9 are separated by water, we would agree that the
two districts are equally compact. The Voter Exchange Index is thus designed to account for the presence of water and state boundaries along the edges of election districts.

We calculated the scores on the Voter Exchange Index for all 435 districts that elected representatives to the 110th U.S. Congress in 2004. We then ranked all districts in increasing order of their scores on that index—from the least compact to the most compact. We also found the norm—the average score on the Voter-Exchange Index for the country—to be 0.70, the highest score to be 1.00, and the lowest score to be 0.02. Figure 7 displays the results of our findings in graphic terms. It includes nine district shapes and their corresponding Equal-Land-Area Circles.

The Voter Exchange Index is a direct and relevant measure of district compactness. Like other measures in the literature, it takes the value of 1 if no voters were rejected from the Equal-Land-Area Circle, and values between 0 and 1 otherwise. If voters in a substantial share of the area of that circle were rejected to create the district, the Voter Exchange Index will have a value close to 0.

The reader can now ascertain that this index overcomes three out of the four critical weaknesses identified in measures that have been previously proposed in the district compactness literature: First, it is a relevant measure of district compactness because it is intimately related to the function of non-compact districts—voter exchange for political gain. In other words, it is able to detect and measure the First Amendment and Fourteenth Amendment harms created by excessive gerrymandering. Second, it is highly correlated with ordinary people’s intuitive perceptions of shape compactness. Third, it takes into
account local geographical constraints such bodies of water and state boundaries, while none of the compactness measures in the redistricting literature account for bodies of water and state boundaries. The remaining question is whether the Voter Exchange Index can *identify and consistently reject* district shapes that are perceived as the most non-compact and the most aggressively gerrymandered by most ordinary people. This issue is taken up in the next and final section of this essay.

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Figure 7: Nine U.S. Congressional Election Districts in 2004, their Corresponding Equal-Land-Area Circles, and their Scores on the Voter Exchange Index (VEI)

Note: The top left shape shows the district with the highest score on the Voter Exchange Index. The top middle one shows the average score, the Voter Exchange Index norm in 2004. The top right one shows the district with the median score, namely the district that had 50% of the districts with higher scores and 50% with lower scores on the Voter Exchange Index. The rest show districts that had the highest scores among districts with the lowest 20% score, the lowest 10% score, the lowest 5% score, the lowest 2% score, and the lowest 1% score. The right-hand shape in the second row shows the district that had slightly more than 50% (a majority share) of its area in the Equal-Land-Area Circle, in other words a score slightly higher than 0.5 on the Voter Exchange Index.

Conclusion: Identifying and Consistently Rejecting Impermissible District Shapes

The emerging conclusion from our earlier discussion is that the central harms from excessive gerrymandering are First Amendment and Fourteenth Amendment harms to voters who were rejected in large numbers from voting in the district nearest them—and replaced by voters that are foraged far away—for no other reason other than their voting history. We suggest that the Supreme Court can adopt an enforceable and justiciable
standard that can mitigate these harms, a standard based on the appropriate and relevant measurement of the compactness of district shapes. We observe that gerrymandering is identical with voter exchange—expelling voters living close by and replacing them with voters living further away for political gain—and that a compactness standard based on a quantitative measure of voter exchange can successfully detect and deter gerrymandering.

Voter Exchange is best measured by a simple index—the Voter Exchange Index. This index compares the shape of the district to a baseline district—the Equal-Land-Area Circle. This is a circle about the geographic center of the district that contains the same land area in the State as that of the district. The Voter Exchange Index measures the share of the land area in the Equal-Land-Area Circle housing voters that still vote in the district.

The authors are of the opinion that it may be advisable not to construct a very high threshold on this index, so as to allow districting officials to meet other districting requirements such as preserving population equality among districts, creating minority-majority districts, preserving established administrative boundaries, or promoting the creation of competitive districts. The main objective of adopting a standard to restrain gerrymandering is to eliminate its most virulent forms, not to straight-jacket the districting process.

There have been many calls for mandating that redistricting be carried out by disinterested parties: blue ribbon commissions, panels of retired judges, or unaffiliated expert groups, insisting that “the Court should forbid ex ante the participation of self-
interested insiders in the redistricting process”¹⁰¹ altogether. Still, the authors suspect that, given the high stakes involved, even if redistricting were done by independent bodies may still be subject to the influence of self-interested insiders. We therefore propose that the Supreme Court adopt a simple compactness standard that will constrain gerrymandering to a substantial degree, whether or not the redistricting process excludes self-interested parties.

More specifically, we propose that the Court adopt the following compactness standard to restrain voter exchange through gerrymandering:

The *Majority Share Standard* requires that voters in the majority share of the area of the Equal-Land-Area Circle of every district vote in the district.

In other words, this standard requires that the Voter Exchange Index of every district in every state be greater than \(\text{one-half}\). In 2004, for example, there were 44 out of the 435 districts (10%) that did not conform to the proposed Majority Share standard. This number was higher than its corresponding number in 1996. In 1996, there were 30 out of the 435 districts (7%) that did not conform to the proposed Majority Share standard.

The Majority Share rule will provide relief for large numbers of voters who were harmed by excessive voter exchange. That said, this standard does not seek to protect a single voter that was rejected from voting in a particular district that was closest to her before it was gerrymandered. Neither does it seek to protect a voter that was brought in from far

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away to vote in a particular district through aggressive gerrymandering. The Court recognizes that redistricting needs to fulfill a number of requirements. Given these requirements, there exists no redistricting plan that ensures that everyone votes in the district they would expect to vote in the absence of gerrymandering. Any quantitative standard the Court chooses to apply may be subject to definitional and measurement errors.

For those reasons, any standard chosen by the Court to restrain aggressive gerrymandering must, of necessity, have a wide margin or error, allowing a State legislature sufficient—while not excessive—latitude in the preparation of redistricting plans. A plaintiff claiming to have been prevented from voting in a particular district that was nearest her before it was gerrymandered must therefore demonstrate that she was one among many voters in a large area near the center of that district—more specifically, a majority share of the Equal-Land-Area Circle associated with that district—that were expelled from the district because of their voting history. If voters like her from the majority share of the area of that hypothetical district were expelled then she can claim with certainty that her First Amendment and Fourteenth Amendment rights were infringed by excessive voter exchange.

The detection of aggressive voter exchange in a district shape using this standard does not help determine whether a particular district was gerrymandered during the latest round of redistricting or in earlier rounds of redistricting. It does not compare the shape of the district to the shape of the district that was there before. It only compares it to its baseline district, a hypothetical district that would have emerged naturally in the absence of
gerrymandering. Neither does this standard seek to unearth the particular motivation for gerrymandering, to determine whether it was racial or political, partisan or bi-partisan, or perpetrated by one or another party. It simply provides a quantitative test for determining whether a particular district shape was excessively gerrymandered for any and all reasons.

Surely, the Court may choose a stricter or a more lenient standard, if it decided that such a standard—in combination with other restraints—was necessary to repair the Constitutional harms created by unchecked gerrymandering. The Court may also consider the political implications of setting the standard too high or too low. In addition, the authors readily agree that—like any proposed standard—the Majority Share standard should be subject to painstaking scrutiny and rigorous further study before being adopted by the Court. Many similar standards have been proposed before and each and every one of them was found to be wanting in one way or another. Young, for example, has demonstrated that any compactness measure “is defective in some respect”, namely that—given any standard of compactness—a shape can be “artfully concocted” that conforms to the standard but that would not be intuitively perceived as compact.102

Young’s insight raises an important question: to what extent is the proposed Majority Share standard subject to error? More specifically, to what extent is it subject to two kinds of error—errors or commission and errors of omission? In other words, does the Majority Share standard reject district shapes that should, in fact, be permissible (an error of

102 Young, H.P., 1988, “Measuring the Compactness of Legislative districts”, Legislative Studies Quarterly 13(1), February, 112.
commission); and does it fail to reject district shapes that should not, in fact, be permissible (an error of omission)?

We can answer the first question by looking, for example, at each and every election district for the 110th Congress in 2004 that was detected by the Majority Share rule and judged to be impermissible. These districts are shown in figure 8 in order of increasing compactness—from the least compact to the most compact—where the last district in the array, Ohio district 13, has exactly half of the area of the hypothetical Equal-Land-Area Circle containing people that still vote in the district. As figure 8 shows, the 44 district shapes that were detected as impermissible by the Majority Share standard in 2004 can all be said to be non-compact and highly gerrymandered. In other words, casual observers will agree that all of these shapes deviate from simple and compact forms.

We can therefore conclude that the Majority Share standard does not appear to contain significant errors of commission: it does not reject district shapes that should, in fact, be permissible. This is an important result because it conforms with our contention that the limits on gerrymandering should be set on the causal factors that lead to non-compact districts—namely, voter exchange—rather than on the effects of voter exchange—namely the appearance of non-compactness. It demonstrates that the appearance of non-compactness need not be regulated. It will disappear by itself if the cause of non-compactness—namely, voter exchange—is subject to proper restraint.

We next examine possible errors of omission: Does the Majority Share standard fail to reject district shapes that should not, in fact, be permissible? Young, for example, could no
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doubt “artfully concoct” a district shape that had just slightly more than half the district area within the Equal Land-Area Circle, arrange for the area inside the circle to be cut up into numerous shapes, and then forage for the rest of the voters as far away from the district as possible. This would again result in bizarre district shape that, although they have a substantial core, may still have created an Expressive Harm by looking far from compact.
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Figure 8: The 44 districts in the 110th Congressional election in 2004 that were rejected as impermissible by the Majority Share standard

Note: Each map shows the district in dark grey, the state in light grey, and the Equal-Land-Area Circle in black. The scores on the Voter Exchange Index (VEI) are shown on the bottom right of each district map.

The proposed Majority Share standard has other limitations as well. It does not detect aggressive gerrymandering at the micro level, for example. Gerrymandering at the micro level is of concern to the Justices in two specific ways. First, when it produces districts that have parts of them connected by very thin strips, creating districts shaped like “dumbbells,” or “[a]t one point the district remains contiguous only because it intersects at a single point with two other districts before crossing over them.” Second, when it produces districts with jagged boundaries that reject voters block-by-block, as in “the


relative smoothness of the district lines made the district compact”\textsuperscript{105}, “a tortured… boundary”\textsuperscript{106}, or “an uncouth twenty-eight-sided figure”\textsuperscript{107} 108

We have examined all the districts in the 2004 election cycle for the subjective appearance of non-compactness. We have identified 31 districts that the Majority Share standard failed to detect that we believe to be non-compact and aggressively gerrymandered. These are shown in figure 9, arranged from top left to bottom right in order of increasing scores on the Voter Exchange index. A cursory examination of figure 9 reveals a number of interesting issues. First, some districts were not identified as impermissibly gerrymandered because they have values on the Voter Exchange index that are just above one-half, the first 4 districts in the top row, for example. Other districts that the Majority Share rule failed to detect are kite-shapes districts—districts with a substantial core and one or more thin tentacles extending out of the core. Still other districts that the


\textsuperscript{108} We used a smoothing algorithm to estimate the extent of voter exchange that can be attributed to serrated, non-smooth, district boundaries. We have come to a preliminary conclusion that it is rather small compared to the extent of voter exchange that results from expelling voters living close to the center of the district and replacing them with voters living further away.
standard failed to identify are those that have cuts into their centers that are very deep, but do not expel voters in too large an area close to the center.

For that reason, we must conclude that the proposed Majority Share rule is free of errors of commission but not of possible errors of omission. In this sense, the proposed rule is an asymmetrical rule. It is a rule that eliminates non-conforming district shapes, but it does not provide safe harbor for district shapes that do conform to the rule. It eliminates most of the offending districts, but it does not condone other districts that conform to the rule but may still be shown to create an Expressive harm as well as other Constitutional harms still to be identified in the future.

There is no question that additional rules can be devised to contain aggressive gerrymandering of all kinds. For now, the authors have only identified one rule—the Majority Share rule—that is simple enough to understand and implement, and that is firmly grounded in the Constitutional harms that are perpetrated by designing districts that fail to adhere to that rule. For the time being, until more and better rules are devised, there is no reason for the Court not to adopt this rule. It eliminates most districts that are aggressively gerrymandered, and, as Justice Souter demands, it is needed “for courts to identify at least the worst cases of gerrymandering and to provide a remedy”\(^\text{109}\).

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Figure 9: 31 highly-suspect districts in the 110th Congressional election in 2004 that were not rejected as impermissible by the Majority Share standard

Note: Each map shows the district in dark grey, the state in light grey, and the Equal-Land-Area Circle in black. The scores on the Voter Exchange Index (VEI) are shown on the bottom right of each district map.

We therefore recommend that the Supreme Court give guidance to the makers of redistricting plans by placing lower limits on the quantity and severity of voter exchange. More specifically, we recommend the Majority Share rule as such a standard, requiring that the majority share of the area of the district be inside a hypothetical circle about the center of the district, a circle with a land area in the state equal to that of the district. Such a clear
and transparent prophylactic standard will reduce the uncertainty involved in the preparation of redistricting plans. It will ensure that even independent commissions will be bound by a rule that restrict their ability to engage in the willful distortion of district shapes for one reason or another. It will not provide a safe haven for those that follow the standard at the present time. But it will start us on a path towards a cure the malaise of unbridled gerrymandering, the malaise that now infects the most basic of our democratic institutions.

* * *

Non-Compactness As Voter Exchange
Annex I: Gerrymandering Is Not ‘A Self-Limiting Enterprise’—

The Empirical Evidence

Indeed, there is good reason to think that political gerrymandering is a self-limiting enterprise. [Justice O’Connor, *Davis v. Bandemer*, 1986]¹⁰

Justice O’Connor, in the quote above, contends that the political gerrymandering is self-correcting and therefore requires no judicial oversight. She continues:

There is no proof before us that political gerrymandering is an evil that cannot be checked or cured by the people or by the parties themselves. Absent such proof, I see no basis for concluding that there is a need, let alone a Constitutional basis, for judicial intervention.¹¹

The question whether or not political gerrymandering is a self-limiting enterprise is an empirical question, not a legal one. And while it is not possible to provide a watertight mathematical ‘proof’, we do offer solid empirical evidence here that it is *not* a self-limiting enterprise. This evidence implies, though it by no means proves, that *there is* indeed a need, and possibly a Constitutional basis, for judicial intervention.

We based our test of Justice O’Connor’s conjecture on three assertions—expounded on in great detail in this essay: first, that *all* gerrymandering is political; second, that


gerrymandering is basically the exchange of voters living ‘close by’ for voters living ‘further away’ for political gain; and third, that the shape of election districts provides us with evidence of ‘illicit purpose’ in drawing up election district maps, more specifically, that district compactness is an expression, or a direct result, of voter exchange. By extension, since there is a direct causal relationship between voter exchange and compactness, if it were shown that district compactness is not self-limiting, then we will have proven that gerrymandering is not a self-limiting enterprise. In other words, if gerrymandering were self-limiting then district compactness would be expected to be self-limiting as well—compactness measures of district shapes would then be expected to fluctuate about a certain average, but would not be expected to decline significantly over time. If, however, compactness measures of district shapes were found to decline significantly over time, we could not but conclude that compactness is not self-limiting, and that therefore gerrymandering is not ‘a self-limiting enterprise’.

There is no reason to believe a priori that the non-compactness of election districts that resulted from partisan gerrymandering by one party will be corrected by partisan gerrymandering by the other party, or by bipartisan gerrymandering by both parties. Pildes and Niemi, for example, who compared the compactness of Congressional districts in 1980s and the 1990s using two simple compactness indices, concluded that “there is no denying that the present congressional districts are less compact that those they replaced.”

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112 Pildes and Niemi, 1993, 573.
We repeated the measurements of Pildes and Niemi for the period 1996-2004, using their two indices as well as ten other compactness indices and came to a similar conclusion:

We can say with perfect confidence that, on average, the compactness of U.S. Congressional districts, measured by twelve different compactness metrics, declined between 1996 and 2004.

The twelve compactness metrics used in our calculations were all based on comparing the district shape—as a blot on a blank background—to the shape of a circle.\textsuperscript{113} They include the best compactness indices found in the literature as well as some new indices defined by the authors, those relevant to measuring election district shapes and those entirely irrelevant for measuring election district shapes. They are:

1. The \textit{Cohesion Index}, defined as the ratio of the average distance among all points in the Equal-Area Circle and the average distance among all points in the shape;

2. The \textit{Proximity Index}, defined as the ratio of the average distance from all points in the Equal-Area Circle to its center and the average distance to the center of the shape\textsuperscript{114} from all points in the shape;

3. The \textit{Spin Index}, defined as the ratio of the Moment of Inertia of the Equal-Area Circle and the Moment of Inertia of the shape about an axis through its Center of Gravity;

\textsuperscript{113} For a fuller exposition of these metrics, see Angel, Parent, and Civco, 2008.

\textsuperscript{114} Defined as the centroid or center of gravity of the shape, the point with the minimum distance-squared to all points in the shape.
4. The Area-Exchange Index is the share of the total area of the shape that it shares with the Equal-Area Circle about its center.

5. The Perimeter Index, defined as the ratio of the perimeter of the Equal-Area Circle and the perimeter of the shape;\textsuperscript{115}

6. The Fractal Index, defined as the ratio of the Fractal Dimension\textsuperscript{116} of the shape and the Fractal Dimension of its Equal-Area Circle;

7. The Fullness Index is the ratio of the average fullness of small neighborhoods along the perimeter of the shape and of the Equal-Area Circle, where a \textit{small neighborhood} around a given point is a circle with an area equal to 1% of the total area of the shape, and \textit{Fullness} is the share of the area of that small neighborhood that belongs to the shape, rather than to its periphery.

8. The Depth Index, defined as the ratio of the average distance to the periphery in the shape and the average distance to the periphery in the Equal-Area Circle;

9. The Dispersion Index, defined as the ratio of the area of the shape inside the Average-Distance Circle and the area of the Average-Distance Circle, where the \textit{Average-Distance Circle} is a circle about the center of a given shape with a radius equal to the average distance of points on the perimeter of the shape to its center.

10. The Range Index of a given shape is the ratio of the diameter of its Equal-Area Circle and the diameter of the smallest circle fully circumscribing the shape;\textsuperscript{117}

\textsuperscript{115} A similar index was used by Pildes and Niemi, 555.

\textsuperscript{116} Defined as twice the logarithm of the perimeter of the shape divided by the logarithm of its area.
11. The **Girth Index** is the ratio of the thickest layer insulating the innermost point of the shape\textsuperscript{118} from its periphery and the radius of the Equal-Area Circle;

12. The **Detour Index** of a given shape is the ratio of the perimeter of its Equal-Area Circle and the perimeter of its Convex Hull, where the Convex Hull of a given shape is the convex polygon of the shortest possible perimeter that fully encompasses it.

Table A1.1 presents the average score on each of the twelve compactness indices for all 435 U.S. Congressional districts in 1996 and 2004. The table shows that we can conclude with perfect confidence that the compactness of U.S. election district shapes—measured on a blank background—declined significantly between 1996 and 2004, no matter how compactness is measured. Since compactness measures of shapes on a blank background best correspond to people’s intuitive perceptions of compactness, as we shall see in annex II below, we can conclude that the *appearance* of compactness of district shapes declined significantly between 1996 and 2004.

<table>
<thead>
<tr>
<th>Compactness Index</th>
<th>Average Compactness Score</th>
<th>% Change 1996-2004</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1996</td>
<td>2004</td>
</tr>
<tr>
<td>Cohesion</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>Proximity</td>
<td>0.81</td>
<td>0.78</td>
</tr>
<tr>
<td>Spin</td>
<td>0.63</td>
<td>0.59</td>
</tr>
<tr>
<td>Area Exchange</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>Perimeter</td>
<td>0.48</td>
<td>0.43</td>
</tr>
</tbody>
</table>

\textsuperscript{117} The Range Index is called the Dispersion Index in Pildes and Niemi, 1993, 554.

\textsuperscript{118} The innermost point of a shape is the point that is furthest away from its periphery.
The table also shows that district compactness declined significantly when we measure it in terms of the Voter Exchange Index. Gerrymandering, measured in terms of voter exchange that takes into account geographical constraints such as bodies of water and state boundaries, also declined significantly between 1996 and 2004.

Both the Pildes and Niemi data comparing the 1980s and the early 1990s and our data comparing 1996 and 2004 suggest a decline in district compactness. Skeptics may argue that these are normal fluctuations in district compactness that average out over the longer term. To examine this claim, we next compared the change in six measures of district compactness over the past two centuries, from 1781 to 2004, for one representative state, the State of New York. More particularly, we compared the average compactness of the three least-compact districts in every decade, starting in 1781. For the years 1781 to 1981 we used the data

We selected the three least-compact districts by visual inspection.
given in Martis.\textsuperscript{120} For 1996 and 2004 we used district shape data from the US Census. The examination of these data leads us to conclude that:

The average compactness of the three least compact districts in New York State was relatively stable between 1781 and 1961 and has been in decline ever since.

As shown in figure A1.1, district compactness in New York State was relatively stable between 1781 and 1961, fluctuating about a fairly stable average. Since 1961, however, it has been in steady decline on all six compactness measures. In other words, we suspect, given the examination of this limited data set, that the average decline observed earlier by Pildes and Niemi between the mid-80s and the mid-90s and by the authors between 1996 and 2004 for the country as a whole actually started earlier. The data for New York State suggest that that average district compactness in the United States has been in decline for almost fifty years. It should not be difficult to examine the available historical data for other states, and we leave this for further study.

Finally, it may be argued that gerrymandering is universal and that non-compact district shapes will be found wherever there are territorially-based elections, and that the average compactness of U.S. election districts will not be very different from the average compactness of election districts elsewhere. To test this claim, we compared the average compactness of U.S. Congressional election districts in 2004 with the average compactness

of election districts in the United Kingdom in 2005, again using the twelve compactness indices defined earlier. The result of this comparison is unequivocal:

We can say with perfect confidence that the average compactness of U.S. Congressional districts in 2004—measured by twelve different compactness indices—was lower than the average compactness of election districts in the United Kingdom in 2005.

![Figure A1.1: Average Compactness Scores on Six Indices in the Three Least Compact Districts in New York State, 1790-2004](image)

Note: Compactness values range from 0 to 1, with 0 indicating the lowest value and 1 the highest.

Table A1.2 presents the average score on each of the twelve compactness indices for U.S. Congressional districts in 2004 and U.K. election districts in 2005. The table shows that the compactness of U.S. election districts in 2004 was considerably lower than the compactness of election districts in the United Kingdom in 2005, no matter how compactness is measured. In fact, it was found to be lower on all measures of compactness.
No other comparisons have been attempted to-date, but it may be rather alarming if the United States, a beacon of democracy, turned out to have very low scores on election district compactness in comparison with other countries. We leave the exploration of these comparisons for further study.

To conclude, our findings show in no uncertain terms that election district compactness in the United States has been in significant decline for a considerable period, that recent declines have been precipitous, and that U.S. Congressional districts are now much less compact than election districts in the United Kingdom. These findings also lead us to suspect, awaiting further study, that the extent and severity of gerrymandering in the U.S. is now well-above the global norm. At the very least, our findings lead to the inevitable conclusion that gerrymandering in the United States—defined as aggressive voter exchange—is on the increase, and that it has been increasing steeply in recent years. The available evidence suggests, therefore, that contrary to Justice O’Connor’s assertion, gerrymandering is not a ‘self-limiting enterprise’.

The steady decline in district compactness since the 1960s is not surprising and may be attributable to two principal factors: changes in legal doctrine and improved technology.\textsuperscript{121} Since \textit{Karcher v. Daggett},\textsuperscript{122} redistricting plans have had to sacrifice compactness to ensure that districts contain exactly equal populations. Since \textit{Thornburg v.}

\textsuperscript{121} See discussion in Pildes and Niemi, 1993, 574-575.

\textsuperscript{122} Karcher v. Daggett, 462 U.S. 725 (1983).
Non-Compactness As Voter Exchange

Gingles,\textsuperscript{123} redistricting plans have certainly had to sacrifice compactness to facilitate the
creation of minority-majority districts. Rapid computation capabilities and Geographical
Information System (GIS) software have transformed gerrymandering into an esoteric art,
enabling its adepts to create tailor-made redistricting plans that maximize voter exchange in
pursuit of the particular political agendas of those who employ them.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Cohesion</td>
<td></td>
<td>0.78</td>
<td>0.86</td>
<td>8.4%</td>
</tr>
<tr>
<td>Proximity</td>
<td></td>
<td>0.78</td>
<td>0.87</td>
<td>8.3%</td>
</tr>
<tr>
<td>Spin</td>
<td></td>
<td>0.59</td>
<td>0.72</td>
<td>13.1%</td>
</tr>
<tr>
<td>Area-Exchange</td>
<td></td>
<td>0.65</td>
<td>0.74</td>
<td>9.1%</td>
</tr>
<tr>
<td>Perimeter</td>
<td></td>
<td>0.43</td>
<td>0.54</td>
<td>10.7%</td>
</tr>
<tr>
<td>Fullness</td>
<td></td>
<td>0.95</td>
<td>0.97</td>
<td>2.1%</td>
</tr>
<tr>
<td>Depth</td>
<td></td>
<td>0.52</td>
<td>0.61</td>
<td>8.8%</td>
</tr>
<tr>
<td>Dispersion</td>
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<td>0.85</td>
<td>0.88</td>
<td>2.8%</td>
</tr>
<tr>
<td>Range</td>
<td></td>
<td>0.58</td>
<td>0.65</td>
<td>7.6%</td>
</tr>
<tr>
<td>Girth</td>
<td></td>
<td>0.54</td>
<td>0.61</td>
<td>6.3%</td>
</tr>
<tr>
<td>Detour</td>
<td></td>
<td>0.67</td>
<td>0.78</td>
<td>10.3%</td>
</tr>
<tr>
<td>Fractal Dim.</td>
<td></td>
<td>0.92</td>
<td>0.94</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

Table A1.2: Variation in the Average Compactness between U.S. Election Districts in 2004 and
United Kingdom Election Districts in 2005

\textit{Note:} Compactness values range from 0 to 1, with 0 indicating the lowest value and 1 the highest.

The absence of any effective legal restraints on the redistricting process may have
emboldened ambitious politicians, giving them license to test the limits of the permissible.
Their opponents could not afford to fall far behind, lest they lose some electoral advantage.

\textsuperscript{123} Thornberg v. Gingles, 478 U.S. 30 (1986).
In short, politicians in both parties have had and continue to have clear incentives and sophisticated tools to engage in the creation of non-compact district shapes in pursuit of their political fortunes, and there are presently no self-correcting incentives nor are there any restraints that will prevent them from creating less and less compact district shapes in the years to come.

* * *

Annex II: The Appearance of District Compactness is Measurable—

The Empirical Evidence

[w]e believe that reapportionment is one area in which appearances do matter. [Justice O’Connor, Shaw v. Reno, 1993]124

The U.S. Supreme Court has recognized that the legitimacy of democratic elections may be compromised if voter confidence in the fairness of these institutions and in the ability of the Constitution to protect them is diminished and harmed by the persistent appearance of district shapes that are not compact.125 The importance of appearance in the adjudication of


125 Pildes and Niemi define this harm as an expressive harm. See Pildes and Niemi, 1993, 485.
gerrymandering cases raises a number of important questions: Do people share perceptions about the appearance of compactness or is it only in the eye of the beholder? What indices of compactness best correspond to common perceptions of the appearance of compactness? Do people need to be trained or assisted in order to correctly detect compactness? Does the appearance of non-compactness itself need to be regulated or will it disappear in and of itself if voter exchange is regulated?

It has been suggested, with reference to Justice Potter Stewart, that compactness is a bit like pornography, we can’t define it, but we know it when we see it. Individual sensitivities, it is claimed, vary widely. The authors set out to dispute this contention, seeking to examine the degree to which there is common agreement among individuals about the appearance of compactness of district shapes. We did this by conducting a pilot survey: giving disinterested volunteers—81 students at Princeton University and at the University of Connecticut—a page with nine district shapes drawn on it (see figure A2.1)


127 See, for example, Monmonier, M., 2001, Bushmanders and Bullwinkles: How Politicians Manipulate Electronic Maps and Census Data to Win Elections, Chicago: University of Chicago Press, 64.

128 The survey was conducted with in close collaboration with Dr. D.L. Civco, Professor of Geomatics at the Department of Natural Resources Management and Engineering at the University of Connecticut.

129 There is no reason to suspect that community standards would vary when it comes to detecting compactness.
and asking them to rank them in order of decreasing compactness. We did not tell respondents that the shapes were election districts, but we did provide them with a dictionary definition of ‘compactness’.

Figure A2.1: Nine District Shapes Used in a Pilot Survey of Perceptions of Compactness, 2007

The Shared Perceptions Finding: People share their perceptions about the appearance of compactness to a very high degree.

We can say with 99% confidence\(^{130}\) that the average compactness ranking people in the general population will assign to any shape (in this survey) will not vary by more than half a rank in any direction. In this particular survey, respondents found it easier to rank compactness with a high level of confidence at some parts of the spectrum, and with a lower

\(^{130}\) The 99% standard error of the average compactness ranking assigned by survey participants to every shape was found to be less than half a rank in any direction from its actual rank in the survey.
level of confidence in other parts of the spectrum where shapes appeared to be more similar. This is illustrated in figure A2.2. In this figure districts were arranged in decreasing order of the average compactness ranking they received. Namely, if a district received a higher average compactness ranking, it was assigned a higher rank, from 9 to 1. The sloping line across the figure marks the points where the average compactness ranking assigned by respondents exactly equals the rank they were assigned. The dark square marks the average compactness ranking assigned by respondents. The empty squares mark the 99% standard errors of the expected spread in the assignment of a particular compactness ranking to the given district shape in the general population.

Figure A2.2: Average Compactness Rankings Found in a Survey of Perceptions of Compactness, 2007
District D was found to be the most compact. It was therefore assigned the rank 9. The average compactness ranking assigned to it by respondents was $8.9\pm0.1$, i.e. very close to 9. Average rankings and spreads that were very close to the assigned rank were also received by districts B, E, I, and G. But the four districts in the middle of the range—C, H, F, and A—had average rankings that were either higher or lower than their assigned rank. District C, for example, had the fifth highest compactness ranking and was therefore assigned the rank 5, but the average ranking it received was only $4.1\pm0.3$. District A had the second lowest compactness ranking and was therefore assigned the rank 2, but the average value it received was $2.9\pm0.4$. This means that if we surveyed again, we may find that the rankings between C, H and F or between H, F, and A will have changed. Namely, people were not so sure in their rankings of compactness in the middle range in this particular survey, but they certainly did agree, to a very high degree, about being unsure. We also note here that district D, the one most resembling a circle, was clearly identified as the most compact, confirming our contention that the circle is the Platonic ‘ideal’ of compactness.

*Four Indices that Detect Perceived Compactness:* The four compactness indices most relevant to the detection of voter exchange—the Cohesion index, the Proximity Index, the Moment-of-Inertia Index and the Area-Exchange Index—are highly correlated with people’s perceptions of the appearance of compactness.

We calculated the scores that each of the nine shapes used in the survey attained on each one of the 12 compactness indices defined in Annex I. We then calculated the correlations
of these scores with the average compactness rankings for each shape by survey participants. The results are displayed in table A2.1. As the table shows, the lowest correlations are with the Range Index, the Dispersion Index, and the Perimeter Index, the most-often-proposed indices in the literature for measuring the appearance of compactness.

The highest correlations are with the Cohesion Index, which measures ‘community’ in the sense of people being close to each other; the Proximity Index and the Moment-of-Inertia Index, both of which measure closeness to the center; and the Area-Exchange Index, which measures the amount of overlap between the district and the Equal-Area Circle about its center.

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<tbody>
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<td>Cohesion</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Proximity</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Moment of Inertia</td>
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<td>0.99</td>
<td>1.00</td>
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<td></td>
<td></td>
<td></td>
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<td>Area-Exchange</td>
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<td>0.98</td>
<td>0.97</td>
<td>1.00</td>
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<td>Perimeter</td>
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<td>Depth</td>
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<tr>
<td>Dispersion</td>
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<td>0.87</td>
<td>0.93</td>
<td>0.83</td>
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<td>1.00</td>
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<td>Range</td>
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<td>0.73</td>
<td>0.63</td>
<td>0.38</td>
<td>0.70</td>
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<td>1.00</td>
<td></td>
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<tr>
<td>Girth</td>
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<td>0.82</td>
<td>0.83</td>
<td>0.80</td>
<td>0.79</td>
<td>0.66</td>
<td>0.97</td>
<td>0.77</td>
<td>0.59</td>
<td>1.00</td>
<td></td>
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<tr>
<td>Detour</td>
<td>0.94</td>
<td>0.93</td>
<td>0.95</td>
<td>0.90</td>
<td>0.82</td>
<td>0.69</td>
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<td>0.76</td>
<td>0.76</td>
<td>0.97</td>
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<td>0.76</td>
<td>0.88</td>
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<tr>
<td>Survey Mean</td>
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<td>0.91</td>
<td>0.92</td>
<td>0.93</td>
<td>0.52</td>
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<td>0.78</td>
<td>0.79</td>
<td>0.59</td>
<td>0.82</td>
<td>0.77</td>
<td>0.59</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table A2.1: Correlations between the Mean Perceived Compactness in the Survey and Scores on 11 Compactness Indices for Nine U.S. Election Districts, 2004

Note: The compactness indices are defined in the text. The table also shows the correlations among scores on various indices.

In table A2.1, four indices: the Cohesion index, the Proximity Index, the Moment-of-Inertia Index, and the Area-Exchange Index are all shown to have high correlations with people’s perceptions of the appearance of compactness, all in excess of 0.91. This suggests
that they can all be used as proxies of people’s perceptions of the appearance of compactness. What distinguishes them from other indices is that they are relevant for measuring compactness because they relate directly to voter exchange. Since they are both relevant and highly correlated with people’s perceptions of the appearance of compactness, they should be the preferred indices for measuring the appearance of district shapes. There is no need, in other words, to present district shapes to ordinary people and to ask them to rank these shapes every time a redistricting plan is examined. We can rely on these four indices to predict how people will assess compactness with very high accuracy.

We note, however, that these indices measure the compactness of shapes on a blank background, not taking into account landscape constraints or the distribution of the voting population. Their usefulness in detecting voter exchange is thus limited. To what extent, we ask ourselves, can the landscape-sensitive index introduced in this essay—the Voter Exchange Index—able to detect the appearance of compactness?

In 2004, for example 152 out of a total of 435 election districts (35%) had scores on the Voter Exchange Index which were identical\textsuperscript{131} to their scores on the Area-Exchange Index. In other words, there were sufficiently far away from bodies of water or state boundaries for their scores not to be affected by either of them. To the extent that scores on the Area-Exchange Index can predict people’s perception of compactness in these districts—and we know that they do—so can the scores on the Voter Exchange Index. This is an important finding. If we could regulate the appearance of non-compactness by restricting aggressive

\textsuperscript{131} With differences in the score less than 0.001.
voter exchange, as we proposed in this essay, the appearance of ‘bizarre’ districts will be regulated indirectly. This is preferable because it is easier to show that voter exchange causes Constitutional harm to individual voters in individual districts, than to show that the appearance of ‘bizarre’ district shapes constitutes such harms.

Finally, it is important for people judging the appearance of district shapes to understand that state boundaries and bodies of water naturally reduce the compactness of districts, even in the absence of gerrymandering. If they understood that to be the case, they could adjust their perceptions, so as to account for the presence of bodies of water and state boundaries. We could assist them in making these adjustments by not simply showing district shapes on a blank background. We could show the district shape in dark grey, for example, areas in the State in light grey, and water bodies and out-of-state areas in white.

We could also show the Equal-Land-Area circle super-imposed on the district shape, and explain that it contains the same land area in the State as that of the district. We could then explain gerrymandering as the distortion of district shapes for political gain, and that, in Justice Stevens’s words, “[a]mong those well-settled principles is the understanding that a district’s peculiar shape might be a symptom of an illicit purpose in the line drawing process.” Then, and only then, we could inquire about the appearance of district shapes, asking people directly which district shapes appear to be the most gerrymandered, or most suspect of illicit purpose.

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As of this writing, we have not conducted such a modified survey. We urge other researchers to take on this challenge. It is quite possible that such a survey will affirm that people can make informed choices in identifying less compact and more compact election districts. Over and above that, we may find that certain aspects of shape are more suggestive of *illicit purpose* than others, leading us to pursue novel measures of district shape compactness that can better detect it than our proposed metric, the Voter Exchange Index.

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